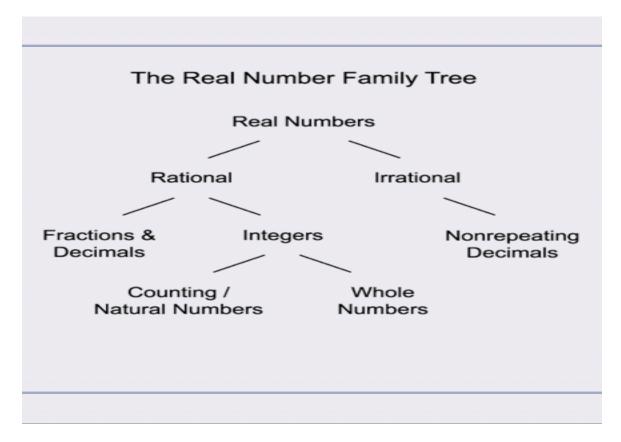
Rules of Algebra (L 001)



(Slide #11)

Order of Operations

Summary

To simplify problems, follow this order:

- 1. Innermost grouping symbols, simplify the expressions within all grouping symbols.
- 2. Simplify terms with exponents.
- 3. Simplify multiplication and/or division in order from left to right.
- 4. Simplify addition and/or subtraction in order from left to right.

(Slide #31)

Khan Academy helpful video: http://www.khanacademy.org/math/arithmetic/multiplication-division/order of operations/v/order-of-operations

Simplify.

$$[21-(5+2)]^2 \cdot 3$$



Type the correct answer, then press Enter.

Real Number Properties (L 002)

Commutative Property of Addition and Multiplication

Let a = 5 and b = 7.

Addition Multiplication

a + b ab is the same as b + a ba

Example: Example: 5 + 7 = 7 + 5 5(7) = 7(5) 12 = 12 35 = 35

(Slide #4)

Associative Property of Addition and Multiplication

If a = 5, b = 6, c = 7

$$a + (b + c) = (a + b) + c$$
 $a(bc) = (ab)c$

$$5 + (6 + 7) = (5 + 6) + 7$$

 $5 + 13 = 11 + 7$
 $18 = 18$
 $5 \cdot (6 \cdot 7) = (5 \cdot 6) \cdot 7$
 $5 \cdot 42 = 30 \cdot 7$
 $210 = 210$

(Slide #5)

Distributive Property

The distributive property is a mixture of both addition and multiplication.

If
$$a = 5$$
, $b = 6$, $c = 7$

$$a(b+c) = a(b) + a(c)$$

$$5(6+7) = 5(6) + 5(7)$$

 $5(13) = 30 + 35$
 $65 = 65$

(Slide #6)

Here are some more properties to learn and remember.

Reflexive	Transitive	Symmetric	
If a = 8	If a = 5, b = 5, c = 5	If a = 3 and b = 3	
a = a	a = b and b = c, then a = c	a = b and b = a	
8 = 8	5 = 5 and 5 = 5, then 5 = 5	3 = 3 and 3 = 3	

(Slide #7)

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify.

$$3a^2 + 4a^2 + 5a^2$$

12a⁶

12a²

12a⁸

Click on the correct answer.

Algebraic Expressions (L 003)

An expression is a mathematical phrase. It does NOT have an = sign. It is just part of a sentence.

This is an expression:

$$4a + 3$$

(Slide #1)

$$(7a + b) - (5a + 4b) - (10a - 2b)$$

This phrase can be simplified by first removing the parentheses.

$$7a + b - 5a - 4b - 10a + 2b$$

Notice that if a parenthesis is preceded by a negative sign, the sign of every number inside the parentheses becomes its opposite when the parentheses are removed.

(Slide #8)

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify this expression.

$$4a^2b + 2ab - (a^2b - 4ab)$$

$$3a^2b + 6ab$$
 $5a^2b + 6ab$ $3a^2b - 2ab$

Click on the correct answer.

Algebraic Equations (L 004)

The difference between expressions and equations is an equal sign (=). An expression is a phrase, but an equation is a complete sentence. Its verb is the equal sign (=).

(Slide #1)

This formula/equation is used often to find the distance traveled.

 $D = r \cdot t$

D = distance

r = rate (speed in mph, mpm or mps)

t = time in hours, minutes or seconds

To find distance, both rate and time must always be expressed in the same unit of time, either hours, minutes, or seconds.

(Slide #3)

Notice:

Parentheses are used to represent the word QUANTITY.

The equal sign (=) represents the word IS.

"n" represents the word NUMBER.

(Slide #11)

You should be able to solve the following problem prior to moving to the Practice Test:

Type the algebraic equation for this statement. Let a represent the unknown number.

Three times a number decreased by fourteen is four.



Type the correct answer, then press Enter.

Solving Equations (L 005)

Refresh on: distributing values, combining like terms, and eliminating fractions

Helpful hint: In practice and mastery tests, if there is not a "variable =" given, *only* put the number as your answer

Remember!

If both sides of the equation are the same when simplified, the answer will always be "all real numbers."

$$2a = 2a$$

When the equation is simplified, and there are numbers on both sides that are not equal, it cannot be an equation so the answer will always be "no solution."

$$0 = 7$$

(Slide #23)

You should be able to solve the following problem prior to moving to the Practice Test:

Solve for the variable.

$$2(3a - 9) = 2a - 10(a - 1)$$

Type the correct answer, then press Enter.

Problem Solving 1 (L 006)

Sample = Keesha made a trip to visit a college she wanted to attend. The college is 310 miles from her home. She averaged 60 mph for most of the way, but ran into construction and was only able to average 40 mph during some of the time. If the trip took Keesha 5.25 hours, how man miles of road were under construction?

Distance = Rate * Time (D = rt) D = rt + rt
$$310 = 60 (5.25 - a) + 40a$$

SUMMARY

- 1. Ask yourself, "What am I trying to find?"
- 2. Get a visual idea of the problem.
- 3. Try your ideas and keep trying!
- 4. Substitute the answer back into the problem to make sure it works.

(Slide #6)

You should be able to solve the following problem prior to moving to the Practice Test:

Leonardo flies from Kansas City to St. Louis in 30 minutes. The air distance is 175 miles. What is the speed of the plane?



Type the correct answer, then press Enter.

Rewriting Formulas (L 007)

In order to find Celsius temperature in this formula, the degrees Fahrenheit must be known.

$$C = \frac{5}{9}(F - 32)$$



(Slide #2)

Khan Academy helpful video: http://www.khanacademy.org/math/algebra/solving-linear-equations-and-inequalities/solving for variable/v/rearrange-formulas-to-isolate-specific-variables

Given the formula a = bc + d, rewrite the formula to solve for c.

$$c=\frac{a-d}{b}$$

$$c = \frac{a+b}{d}$$

$$c = \frac{a}{b - d}$$

$$c = \frac{a}{b+d}$$

Click on the correct answer.

Solving & Graphing (L 008)

Domain = x coordinate; Range = y coordinate Slope-intercept form of an equation y = mx + b where m = slope; y-intercept (denoted as "b") is where the line crosses the y axis, or you could also say that it is what the value of y is when the value of x is zero

A line that moves to the RIGHT after it moves up has a POSITIVE SLOPE.

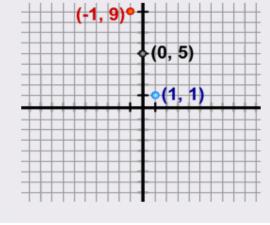
A line that moves to the LEFT after it moves up has a NEGATIVE SLOPE, and the coefficient of x will be negative.

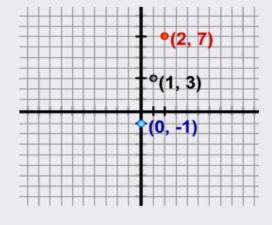
(Slide #11)

You should be able to solve the following problem prior to moving to the Practice Test:

Click on the graph of this equation.

$$y = -4x + 5$$





Properties of Inequality (L 009)

A < B and A < C then A < C Inequality signs do **not** change when adding or subtracting to solve an inequality.

The ONLY time that the inequality in the answer changes (from < to > or from > to <) is when both sides of the inequality are multiplied or divided by a NEGATIVE number.

(Slide #11)

Khan Academy helpful video:

http://www.khanacademy.org/math/algebra/linear inequalities/inequalities/v/one-step-inequalities-2

You should be able to solve the following problem prior to moving to the Practice Test:

Type < or > to make this statement true.



Type the correct answer, then press Enter.

Inequalities (L 010)

Note the difference between a closed circle and an open circle when graphing on a number line.

**Beware of the "Empty Set"

Two mathematical statements separated by and indicate a conjunction or an intersection.

Example: a > 4 and a < 6

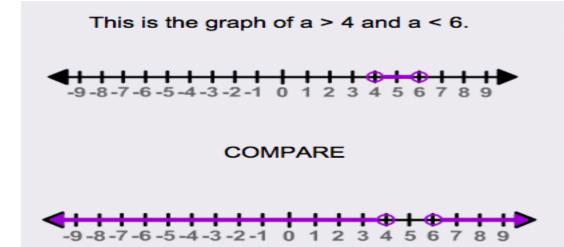
Two mathematical statements separated by or indicate a disjunction or union.

Example: a < 4 or a > 6

(Slide #8)

Khan Academy helpful video:

http://www.khanacademy.org/math/algebra/linear inequalities/compound absolute value inequali/v /compund-inequalities



This is the graph of a < 4 or a > 6.

(Slide #11)

This is another way of writing a conjunction. Study the solution and graph.

$$8 < 2(a + 3) < 10$$

$$4 < a + 3 < 5$$

This time a must be in the middle by itself. To get rid of the 2, divide all three sides by 2.

Subtract 3 from the left side, middle, and right side.

Graph 1 < a < 2 on your paper.

(Slide #20)

You should be able to solve the following problem prior to moving to the Practice Test:

Choose the correct answer.

$$-14 \le 3x - 2 < 7$$

$$-4 \le x < 3$$

$$4 \le x < 3$$

$$4 \ge x > 3$$

$$-4 \ge x > 3$$

Absolute Value Equations (L 011)

Absolute value is the distance between a point on the number line and 0. **All absolute values coming out of the symbol will be positive. If it comes out negative, the answer is NO SOLUTION. When solving an absolute value equation: 1) Isolate the absolute value part; get rid of everything outside 2) You will create two formulas. Formula one is made by simply getting rid of the absolute value symbols. Formula two is made by getting rid of the absolute value symbols and then changing the "number" side of the equation to a negative. **Always** check back your two possible answers. If they both do not work, it is NO SOLUTION.

Let's put a number inside the absolute value with a variable.

$$|2a| = 12$$

This means the product inside the absolute value symbols must equal 12 or -12.

Make the product of 2a equal to 12 and -12. Then solve for "a."

(Slide #10)



Watch the negative signs!

|3a + 4| = -3 is not possible

-4|3a + 4| = -12is possible, because dividing both sides by -4 equals |3a + 4| = 3



Absolute value is ALWAYS a positive real number.

(Slide #12)

CAUTION:

Although there might be at least two possible solutions to an equation, that does not mean both solutions will work. Nor does it mean that there will be two different real numbers.

Be sure to mentally check the ORIGINAL equation to see if your answer makes sense.

(Slide #15)

Khan Academy helpful video: http://www.khanacademy.org/math/algebra/solving-linear-equations-and-inequalities/absolute-value-equations/v/absolute-value-equations

Solve. There are two solutions to this problem.

$$3|2a - 5| = 21$$

Type the correct answers, then press Enter. Follow this example: 2 or -2

Absolute Value Inequality (L 012)

Solving for an absolute value inequality is really not much different from solving for an equation...with one very important difference. You still create two equations as you did before. The difference comes in the second equation. You *not only* have to change the "number" side to negative but you *also* flip the inequality symbol.

$$|4a| < -8$$

The two solutions to this inequality are a < -2 and a > 2.

BUT the "and" means the graph is between the numbers. This is a conflicting statement. Therefore, this solution is NOT possible. The answer is "No Solution."

(Slide #8)



If the absolute value of an inequality is less than (<) a negative number, the answer will always be "no solution."

If the absolute value of an inequality is greater than (>) a negative number, the answer will be "all real numbers."

(Slide #9)

Khan Academy helpful video: http://www.khanacademy.org/math/algebra/solving-linear-equations-and-inequalities-example-3

Follow the steps for solving this inequality.

$$|a+2| < 9$$
 $a+2 > -9$ and $a+2 < 9$
 $a > -11$ and $a < 7$
or
 $-11 < a < 7$

(Slide #10)

You should be able to solve the following problem prior to moving to the Practice Test:

Select the best answer.

$$a \ge -1$$
 and $a \le -11$

$$a \ge -16$$
 and $a \le 4$

$$a \ge -11$$
 and $a \le -1$

Problem Solving 2 (L 013)

<	<	>	>
Less than	At the most	Greater than	At the least
Fewer than	Greatest value	More than	Least value
	No more than		No less than
	Maximum value		Minimum value

Be sure to use care when you encounter these words:

"AND" means ONLY the numbers that are in both sets or equations.

"OR" means either one set of numbers OR another set of numbers. These two sets of numbers could overlap and therefore include ALL REAL NUMBERS in an equation.

(Slide #12)

Number of doses =
$$\frac{\text{total amount}}{\text{size of dose}}$$

(Slide #22)

You should be able to solve the following problem prior to moving to the Practice Test:

Solve this problem.

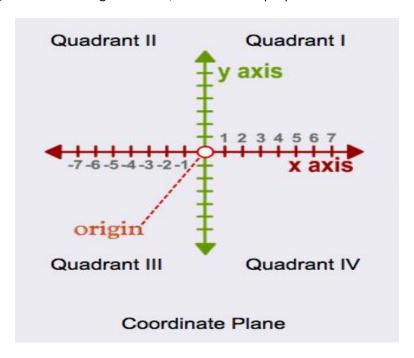
A student needs at least 80% of all points on the tests in a class to get a B. There are three 100-point tests and a 250-point final. The student's test scores are 72%, 85% and 78% for the 100 point tests. What is the least score the student can make on the final and still get a B?



Type the correct answer, then press Enter.

Relations & Functions (L 014)

Hint: When listing domain and range amounts, be sure to use proper braces and do not repeat values.



(Slide #1)

This is an example of an ordered pair. An ordered pair indicates a point on the coordinate plane.

2 has many names:

3 has many names:

x-coordinate

y-coordinate

abscissa

ordinate

domain

range

(Slide #5)

A relation is a set of ordered pairs. Relations can be represented on a graph as a set of ordered pairs.

(Slide #11)

Domain of this relation = $\{-4, -2, 1, 5\}$

Range of this relation = {0, 5, 1, -2}

(Slide #14)

A function is a relation in which the domain is not repeated in another set of the ordered pairs.

In a graph, a quick check to determine if a relation is a function is to draw vertical lines anywhere on the graph. If all of the vertical lines cross the x-axis only once, it is a function.

(Slide #17)

Khan Academy helpful videos:

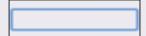
http://www.khanacademy.org/math/algebra/algebra-functions/relationships_functions/v/relations-and-functions

AND

http://www.khanacademy.org/math/algebra/algebra-functions/classic-function-videos/v/introduction-to-functions

Is this relation a function?

$$(-3, 2), (2, -4), (2, 6), (-3, -5), (0, -3)$$



Type yes or no, then press Enter.

Graph Linear Functions (L 015)

The standard form of a linear equation is

$$ax + by = c$$

where a, b, and c are real numbers.

Either a or b, but not both, can be zero.

(Slide #2)

Another form of a linear equation is called the slope-intercept form:

$$y = mx + b$$

where m is the slope of the line and b is the y-intercept.

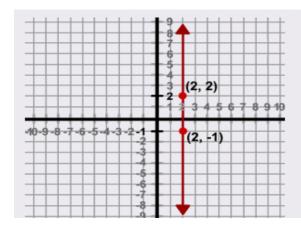
Remember: the y-intercept is the place where the line crosses the y-axis.

(Slide #3)

Hint: when the y-coordinates are the SAME, the equation will be y = y-coordinate.
In this case: y = 3.

Since this graph passes the vertical line test, it is a function. It is a constant function because the y-coordinates are all the same, in this case, 3.

(Slide #11)



The slope for a vertical line is undefined.

Since there is no m and the line does not cross the y-axis, the slope intercept form cannot be written in this case.

(Slide #12)

This line only has an x-intercept.

The equation of a vertical line will be x = x-intercept.

The equation of this line is x = 2.

(Slide #13)

The slope of an equation in standard form can easily be found by solving for y.

$$ax + by = c$$

Let's solve for y.

$$by = -ax + c$$

$$y = \frac{-Ax}{B} + \frac{C}{B}$$

(Slide #22)

This shows that the m, or slope, of an equation in standard form is $\frac{-A}{B}$ and b, or the y-intercept, is $\frac{C}{R}$.

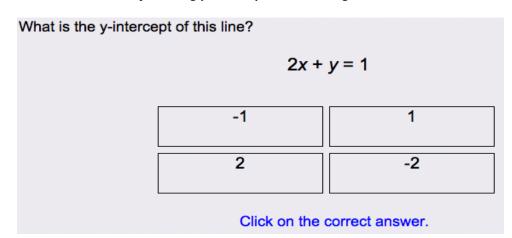
(Slide #23)

We have looked at how to find the slope of a line from an equation. Finding the slope of a line from two points is also possible.

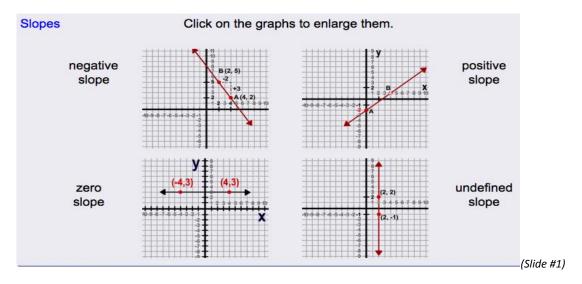
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1, y_1) is one point on the line and (x_2, y_2) is another.

(Slide #27)



Slope of a Line (L 016)



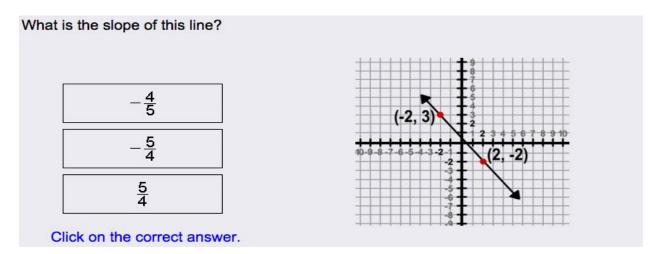
Converting Slope to a Number

$$m = \frac{rise}{rur}$$

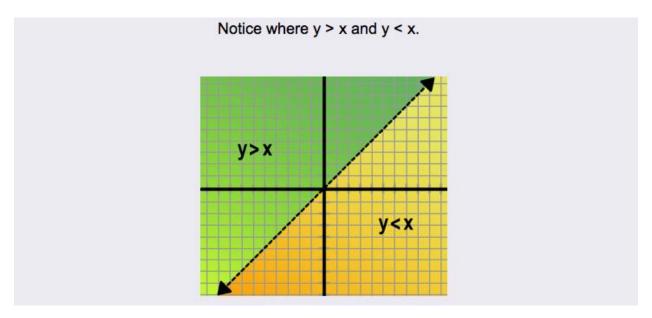
 $m \ = \ \frac{\text{difference of the y-coordinates of two points}}{\text{difference of the x-coordinates of the same two points}}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
 $x_1 \neq x_2$

(Slide #7)



Graph Linear Inequalities (L 017)



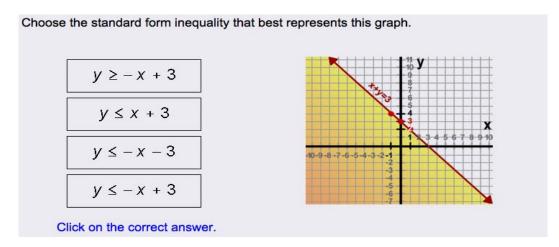
(Slide #3)

Khan Academy helpful video:

 $\frac{http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-linear-inequalities/v/graphing-linear-inequalities-in-two-variables-example-2$

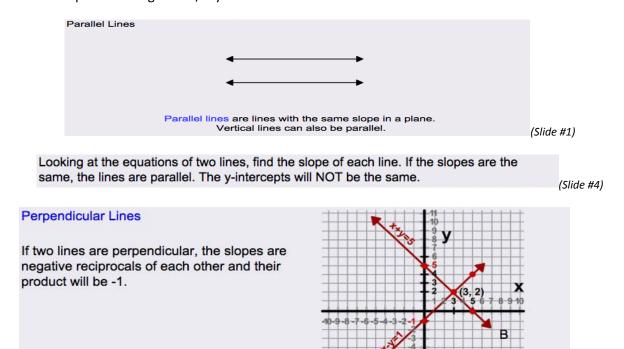
AND

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-linear-inequalities/v/graphing-linear-inequalities-in-two-variables-3



Parallel & Perpendicular (L 018)

Parallel lines have an identical slope but a different y-intercept. Perpendicular lines are multiplicative inverses of each other meaning that when multiplied together, the product will be a -1. We simply flip the first slope and change the +/- symbol to achieve this.



Khan Academy helpful video:

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/perpendicular-line-slope

(Slide #13)

What type of lines are these? $x + 2y = 7 \quad \text{and} \quad 2x + 4y = 21$ $\begin{array}{c} \text{perpendicular} \\ \text{parallel} \\ \text{neither} \end{array}$

Identify Linear Equations (L 019)

Summary of linear equation forms:

slope-intercept form y = mx + bstandard form

ax + by = c

Lines may be written in these forms:

vertical lines x = x-intercept

horizontal lines y = y-intercept

(Slide #2)

(Slide #1)

By using a given point (X_1, y_1) and a slope m, another line form can be developed by letting the unknown point be (x, y).

$$m = \frac{y - y_1}{x - x_1}$$
 (Slide #3)

$$(y - y_1) = m(x - x_1)$$

where $y - y_1 = m(x - x_1)$
point-slope form

When given slope and y-intercept, use slope-intercept form.

When given a point and the slope, use point-slope form.

When given a vertical line, use the x-intercept form.

When given a horizontal line, use the y-intercept form.

(Slide #8)

Equations that represent a function are often written using "f(x) =" instead of "y =."

f(x) is read "f of x." f(x) means the value of x will also affect the value of y.

(Slide #11)

Example:

$$f(5) = 3x - 1$$

The next step is to find the corresponding y value by solving the equation.

This means that an ordered pair located on this line, which is a function, happens to be (5, 14).

(Slide #16)

g[f(x)] is read "g of f of x."

To simplify this, work from the inside out.

$$f(x) = x^2 \text{ and } g(x) = x + 2$$

Find g[f(3)].

(Slide #19)

$$f(x) = x^2$$
 and $g(x) = x + 2$. Find $g[f(3)]$.

Working from inside to out, start with

$$f(3) = (3)^2 = 9.$$

Since f(3) = 9, replace all future f(3) with 9.

Next,
$$g(x) = x + 2$$
.

$$g[f(3)] = g(9) = 9 + 2 = 11$$

$$G[f(3)] = 11$$

(Slide #20)

Find g[f(5)] if f(x) = x + 1 and g(x) = 3x - 2. $g[f(5)] = \boxed{ }$

Type the correct answer, then press Enter.

Problem Solving 3 (L 020)

(Copy all examples; handshakes = subtract 1 from number and add all the way to 0 or use the formula

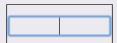
Draw it out if you do not understand.

The equation is
$$y = (\frac{1}{2})x [x - 1]$$
.

(Slide #6)

You should be able to solve the following problems prior to moving to the Practice Test:

Type the equation that shows the relationship between the variables in this chart.



Type the correct answer, then press Enter.
Use slope-intercept form. (y = mx + b)

x	у
0	5
1	4
2	3
3	2
4	1

Type the equation that shows the relationship between the variables in this chart.



Type the correct answer, then press Enter.
Use slope-intercept form. (y = mx + b)

x	У
1	7
2	14
3	21
4	28
5	35

Direct Variation (L 021)

Refresh yourself on how to do cross multiplication

$$\frac{\mathbf{y}_1}{\mathbf{x}_1} = \frac{\mathbf{y}_2}{\mathbf{x}_2}$$

The first and fourth terms are the extremes. The second and third terms are means.

The product of the means equals the product of the extremes.

(Slide #10)

You should be able to solve the following problem prior to moving to the Practice Test:

Are these ordered pairs in direct variation?

{(-1, 0), (3, 2), (9, 6)}

no

yes

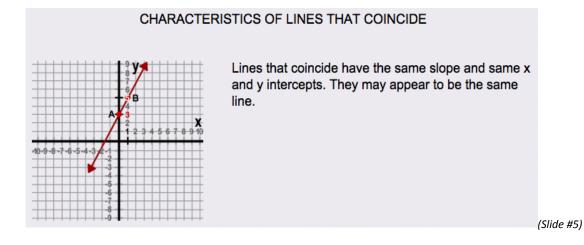
need more facts

Click on the correct answer.

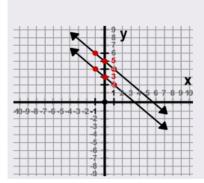
Graphing Equation Systems (L 022)

Coinciding lines = lines that are essentially the same line; they have the same slope and same y-intercept. Parallel lines = lines that have the same slope but a different y-intercept

Perpendicular lines = lines that have negative reciprocal slopes (flipped and change of +/-); multi. to = -1

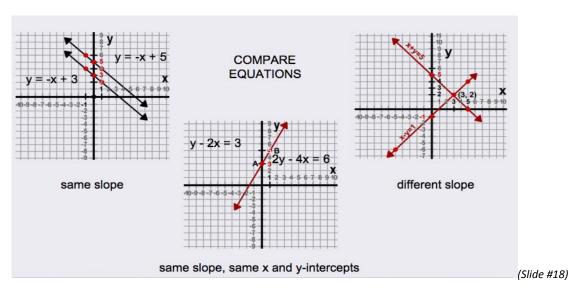


CHARACTERISTICS OF PARALLEL LINES



Parallel lines have the same slope, but different x and y intercepts.

(Slide #8)



You should be able to solve the following problem prior to moving to the Practice Test:

Choose the possible number of solutions for these equations.

$$x - 3y = -3$$

 $-2x + 6y = 12$

no solutions

infinite number of solutions

one solution

Click on the correct answer.

Graphing Systems (L 023)

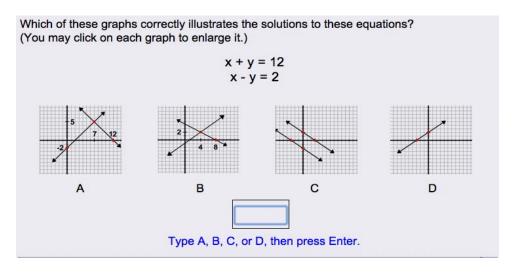
If lines are **not** parallel or coinciding, we know that the two lines have **one solution.**

If lines are parallel, they essentially never touch so there would be **no solution.**

If lines are coinciding, they essentially are the same line and have an infinite number of solutions.

***When graphing these lines, we can make an estimate of the number of solutions and what those solution(s) are. Algebraically, you can find the exact answer and number of solutions (Lesson 24).

You should be able to solve the following problem prior to moving to the Practice Test:



Addition & Substitution (L 024)

Make the decision of what is the most efficient way to solve your system of equations by a) addition method or b) substitution method.

It is best to evaluate the two equations to see which method would require the least amount of work.

(Slide #2)

The **addition method** is best used when you can add the two equations together, and that will result in eliminating one variable. In other words, if I have a positive number of a variable (example "+3y") and add that to the other equation with a negative amount of the same variable (example "-3y"), the result will eliminate that variable completely (example "0y"). Solve for the only variable that is left. Then plug that amount into either equation (whichever looks easiest; i.e. – equation with smaller numbers or without negatives) for that variable to solve for the variable that was initially eliminated. **Sometimes, however, they do not conveniently match up. You will then need to manipulate one or more of the equations by multiplying every term so that the numbers *do* match and cancel out.

Problem

At the park, Joanne and Maria bought food and lemonade for themselves and their friend, Brian. Joanne bought 3 slices of pizza and 1 lemonade for \$4.50. Maria bought 2 slices of pizza and 2 lemonades (one for her and one for Brian) for \$4.00.

How much does Brian owe Maria for his lemonade?





Note that this problem is really asking: "What is the cost of one lemonade?"

(Slide #13)

The **substitution method** is best used when one of the variables is already isolated or can easily be isolated by making one or two quick steps. Once the variable is isolated take what that variable is equal to and substitute that into the other equation. What have you done? Just like the addition method, you have eliminated one of the variables so you can solve for the other variable. Once that is done, you then solve for the variable that was eliminated by plugging the value of the variable that you found into either equation. However, make your life easier by plugging that value into the equation where the eliminated variable was isolated.

There are three possible answers that can occur when solving systems of equations by addition or substitution.

- 1. An answer for both x and y, meaning the lines are intersecting and have only one possible solution.
- 2. Operations used to solve the equations eliminate x and y, resulting in an equation such as 2 = 2. This means that the lines coincide and infinite solutions are possible.
- 3. Operations used to solve the equations eliminate x and y, resulting in equations such as 9 = 4. This would indicate that the lines are parallel and have no points in common.

(Slide #24)

Khan Academy helpful videos:

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination

AND

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-linear-systems-by-substitution

Type the ordered pair that is the solution to these equations. 2x - 3y = 7 -3x + y = 7Type the answer in the box and then press Enter.

Remember to put ordered pairs in parentheses.

Example: (a, b)

Type the ordered pair that is the solution to these equations.

x = 5y 2x - 3y = 14

Type the answer in the box and then press Enter. Remember to put ordered pairs in parentheses. Example: (a, b)

Solving Inequalities (L 025)

Refer back to lesson #17 titled "Graph Linear Inequalities" on proper shading of linear inequalities. This time we are doing the same thing with the exception that we have two equations. The area that they **share** is the solution to our problem. Be very careful when solving your inequalities and putting them in slope-intercept form. Recall that inequality signs flip when you multiply or divide an equation by a negative number. Take special note of what the graph of an absolute value equation looks like. Less than (<) absolute value equations shade *inside*; Greater than (>) absolute value equations shade *outside*.

When solving inequalities on a graph, put the inequality in y = mx + b form.

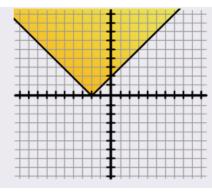
Next, check where the line crosses the y-axis.

If the inequality has a \geq symbol, shade above the line. If the inequality has the \leq symbol, shade below the line.

(Slide #15)

Now, notice what happens when a number is added inside the absolute value.

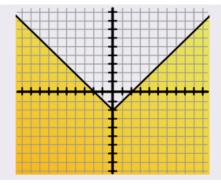
$$y \ge |x+2|$$



(Slide #19)

Compare what happens when a number is subtracted outside the absolute value.

$$y \leq |x| - 2$$



(Slide #21)

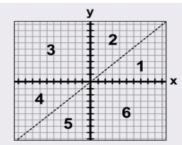
|x| or |y| on the left side of an absolute value inequality < or \le is shaded between the lines.

|x| or |y| on the left side of an absolute value inequality > or $\,\geq\,$ is shaded on opposite sides of each line.

(Slide #30)

You should be able to solve the following problem prior to moving to the Practice Test:

On a piece of graph paper, graph these lines.



Choose the region or regions that form the solution set of the above inequalities.

1 and 6

1, 2, and 6

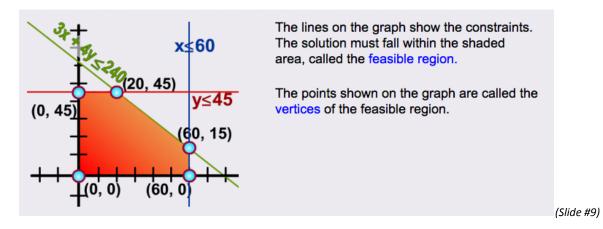
1, 5, and 6

1

Click on the correct answer.

Linear Programming (L 026)

Substitute both values of all vertex points into the equation. Based on all of your results, determine which is optimal depending if you are looking for minimum or maximum values. Note: A+ software is the looking for the total numerical value *not* the coordinate point.



Define the problem.

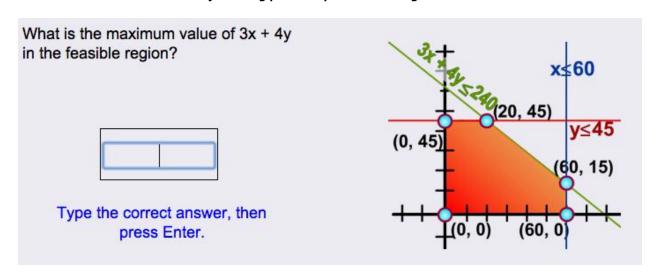
Define the variables and write them in equation form.

Graph the system.

The graphs will form a polygon. Determine the vertices of the polygon. The maximum and minimum points will always be at a vertex. Therefore, substitute the values from each vertex into the equation to determine which point is a maximum (or minimum, if that is what is asked).

(Slide #27)

You should be able to solve the following problem prior to moving to the Practice Test:



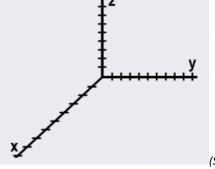
Three-Variable Equations (L 027)

<u>Standard form of a three-variable linear equation</u> Ax + By + Cz = D The *ordered triple* coordinates are (x, y, z).

Solve three equations by the addition method as you have previously done. You will eliminate one variable at a time matching variables as you did before. Once you are down to two variables, keep manipulating the equations until you are down to solving for just one variable. Solve for that variable. Then, plug that value back into two of the equations and go over the whole process again until you solve the remaining two variables.

A linear equation with three variables is written as ax + by + cz = d, where a, b, c, and d are constants.

Example: 4x - 2y + 6z = 8



(Slide #5)

If you get three points and these three points will NOT check out in ALL three equations, the answer is NO SOLUTION.

(Slide #23)

Solve these equations for the ordered triple. It may be necessary to work on a piece of paper.

$$4x + y + z = 7$$

 $x + 3y - z = 0$
 $5x - 3y + z = -6$

(Slide #25)

Khan Academy helpful video:

http://www.khanacademy.org/math/trigonometry/systems_eq_ineq/fancier_systems_precalc/v/systems_of-three-variables

AND

http://www.khanacademy.org/math/trigonometry/systems eq_ineq/fancier_systems precalc/v/systems-of-three-variables-2

Which of these ordered triples indicates where the plane cuts the y-axis for this equation?

$$2x + 3y + 6z = 54$$

(0, 27, 0)

(0, 9, 0)

(0, 6, 0)

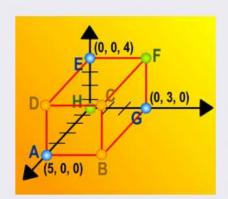
(0, 18, 0)

Click on the correct answer.

Type the ordered triple of point D.



Type the correct answer, then press Enter. (Be sure to put coordinates in parentheses.)



Data in Matrices (L 028)

A matrix is a rectangular arrangement of objects.

Data stored between the brackets are called elements. The data may be read by reading across the rows and reading up and down in columns.

(Slide #1)

Column 1 Column 2

The elements are 7, 4, 3, and 6.

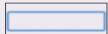
[7 4] Row 1
3 6] Row 2

(Slide #2)

The dimensions of a matrix are the number of rows multiplied by the number of columns.

(Slide #5)

Which element is in the first row, third column of this matrix?



Type the correct answer, then press Enter.

Matrix Multiplication (L 029)

Take very diligent notes on the example for matrix multiplication starting on slide 10. You will build off of matrix multiplication, so it is of the utmost importance that you understand this concept fully.

When multiplying matrices, the number of columns in one matrix must equal the number of rows in the other.

Example:

3 X 4

and

4 X 5

Unless these two numbers are the same, the matrices cannot be multiplied.

(Slide #1)

Now, let's look at how the dimensions of the new matrix are defined after two matrices are multiplied together.

The numbers in blue give the dimensions of the matrix formed after multiplication.

The product of a 3 X 4 matrix and a 4 X 5 matrix is a 3 X 5 matrix.

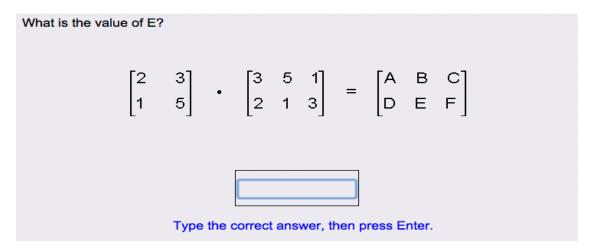
(Slide #3)

Now we have multiplied a 2 X 2 matrix by a 2 X 3 matrix.

$$\begin{bmatrix} -2 & 8 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 30 & 36 & 42 \\ 17 & 22 & 27 \end{bmatrix}$$

The dimensions of the product are 2 X 3.

(Slide #17)



Size & Reflections (L 030)

Calculating Reflections

If the y-axis is the reflecting line and if point A is (x, y), its reflection image has the opposite first coordinate but the same second coordinate. Point A' is (-x, y).

(Slide #16)

If the x-axis is the reflecting line and if point A is (x, y), its reflection image has the opposite of the second coordinate but the same first coordinate. Point A' is (x, -y).

(Slide #20)

If a line y = x is the reflecting line and if point A is (x, y), its reflection image reverses the first and second coordinates.

Point A' is (y, x). (Slide #24)

Reflection Summary

When the reflection is over the y-axis and the x-coordinate is the opposite sign, multiply by this matrix:

(Slide #28)

Reflection Summary

When the reflection is over the x-axis and the y-coordinate is the opposite sign, multiply by this matrix:

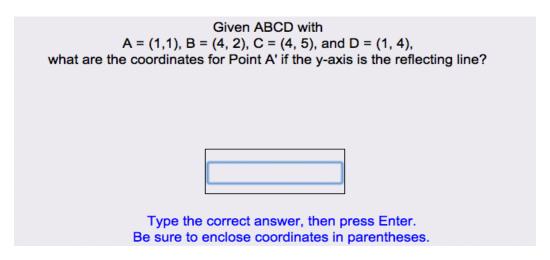
(Slide #29)

Reflection Summary When the reflection is over the line y = x, multiply by this matrix: (Slide #30)

Khan Academy helpful video:

http://www.khanacademy.org/math/linearalgebra/matrix transformations/lin trans examples/v/linear-transformation-examples--scaling-andreflections

You should be able to solve the following problem prior to moving to the Practice Test:



Transformation (L 031)

Transformation is a one-to-one correspondence between sets of points. Size change and reflection is an example of this. Matrices are not always commutative. i.e. Matrix A x Matrix B = Matrix C does not always mean that Matrix B x Matrix A = Matrix C).

They are associative. $(A \times B) \times C = A \times (B \times C)$

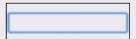
Multiplication is commutative with some 2 X 2 matrices but is not with others.

Multiplying matrix A by matrix B does not always give the same results as multiplying matrix B by matrix A.

(Slide #5)

Summary In multiplication the set of 2 X 2 matrices is: closed, not commutative, associative, has an identity. (Slide #16)

A triangle has points A(1, 2), B(1, 6), and C(3, 6) and is reflected over the x-axis and then over the line y = x. A"B"C" are the coordinates of the reflected image. What is the B" coordinate?



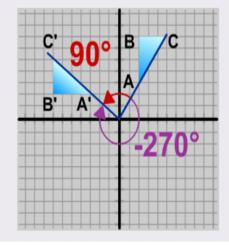
Type the correct answer, then press Enter. (Be sure to enclose coordinates in parentheses.)

Rotation (L 032)

Counter clockwise rotations have positive magnitudes. So 180° represents a -180° counterclockwise turn.

Clockwise rotations have negative magnitudes.

90° represents a counterclockwise turn and has the same image as a rotation of -270° .



(Slide #2)

matrix of rotation for
$$90^{\circ} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

(Slide #5)

matrix of rotation for
$$180^{\circ} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$
(Slide #6)

matrix of rotation for
$$270^{\circ} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

(Slide #7)

What is the image of point (2, 3) if the rotation is 90°?



Type the correct answer, then press Enter. Be sure to enclose coordinates in parentheses.

Matrix Addition (L 033)

To add real numbers in matrices where A, B, and C represent the matrices:

$$A + B = B + A$$

Commutative Property of Addition

and

$$A + (B + C) = (A + B) + C$$

Associative Property of Addition

(Slide #2)

To subtract numbers in matrices:

The difference will be the difference of the elements, but A - B will not equal B - A unless A = B.

(Slide #3)

To add or subtract matrices, both matrices MUST have the SAME dimensions.

(Slide #4)

You should be able to solve the following problem prior to moving to the Practice Test:

Solve for the given element.

Second row, first column. Solve for A - B.

$$A = \begin{bmatrix} 7 & 0 \\ 5 & 11 \end{bmatrix} \qquad B = \begin{bmatrix} 9 & 18 \\ 1 & 10 \end{bmatrix} \qquad C = \begin{bmatrix} 25 & 17 \\ 12 & 31 \end{bmatrix}$$

Type the correct answer, then press Enter.

Exponents (L 034)

You can simply add the exponents if you have the same variable. Look at slide 4 for a trick with fractions that could be helpful. When a variable with an exponent is divided by the same variable with an exponent you can subtract the bottom exponent from the top exponent. If the larger exponent is on the bottom it simply becomes a fraction.

Compare
$$(a^2)(a^3) = (a^2)^3$$
 again.

 $(a^2)(a^3)$ has an "a" (or base) with each exponent. Therefore, exponents must be added.

 $(a^2)^3$ has only one "a" (or base). There are no bases to combine. Therefore, exponents must be multiplied.

(Slide #13)

WHY?

$$\frac{(5a^3b^2)^3}{(10a^4b^5)^2} \,=\, \frac{125a^9b^6}{100a^8b^{10}} \,=\, \frac{5a}{4b^4}$$

Remember, when simplifying fractions, reduce numbers and subtract exponents. Looking at the original problem, put the simplified variable where the largest exponents were located, either numerator or denominator.

(Slide #20)

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify this expression. $(4a^2b^7)^3$ $12a^6b^{21} \qquad 12a^5b^{10} \qquad 64a^6b^{21}$ Click on the correct answer.

Polynomial Types (L 035)

MONomial is ONE term. 3a

term term

Blnomial is TWO terms. 3a + 8b

term term term

TRInomial is THREE terms. 4x + 5y - 9

(Slide #6)

The degree of a term is the sum of its exponents. The degree of a polynomial is determined by the term with the highest degree.

(Slide #10)

$$a^5b^6 - a^6b^8$$

Adding the exponents of the variables in the first term is 5 + 6 = 11. Adding the exponents of the variables in the second term is 6 + 8 = 14. So the degree of this polynomial (binomial) is 14.

(Slide #13)

When working with polynomials, arrange the terms in descending or ascending order by the exponents of a single variable, usually the first variable.

(Slide #17)

Khan Academy helpful video:

http://www.khanacademy.org/math/trigonometry/polynomial_and_rational/polynomial_tutorial/v/add ition-and-subtraction-of-polynomials

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify this polynomial.

$$a^2 - 3ab + b^2 + 2a^2 + 3ab - 2b^2$$

$$3a^2 - 2b^2$$

$$3a^2 - b^2$$

$$3a^2 - b^2 - 6ab$$

Click on the correct answer.

Polynomial Operations (L 036)

Polynomial Operations

When adding or subtracting polynomials, add or subtract only the coefficients of like terms.

NEVER change the degrees of the exponents.

(Slide #1)

Check your answer with this one.

$$(3a - 2b + c) - (7a - 8b + 9c)$$

 $3a - 2b + c - 7a + 8b - 9c =$
 $-4a + 6b - 8c$

(Slide #15)

Perfect Square Trinomials

The perfect square is a special case, because the same binomial is multiplied by itself. This can be done in three easy mental steps or your choice of multiplying two binomials (a + b)(a + b).

(Slide #22)

Look at $(a + b)^2$ using a shortcut.

First step: square the first term = a^2

Second step: multiply first term by second term times 2 (using the sign between the two terms) = +2ab

Third step: square the last term = b^2

Put the three steps together: $a^2 + 2ab + b^2$

(Slide #23)

Difference of Two Squares

Here is another special case. Notice that two binomials can be alike but have opposite signs between the variables.

$$(a + b)(a - b)$$

(Slide #26)

$$(a + b)(a - b)$$

When the variables have opposite signs, the middle term will always cancel out. So all that must be done is to square the first and last terms and add a negative sign between them.

$$(a + b)(a - b) = a^2 - b^2$$

(Slide #27)

Perfect Square Trinomials

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Difference of Two Squares

$$(a + b)(a - b) = a^2 - b^2$$

(Slide #28)

You should be able to solve the following problem prior to moving to the Practice Test:

Solve.

$$(4a - 2b) - (6a - 8b)$$



Type the correct answer, then press Enter.

Factoring Quadratics (L 037)

To factor a quadratic equation remember the FOIL Method.

$$ax^2 + bx + c$$

There are three terms in this expression; therefore, it can be called a quadratic trinomial.

The ax² is called the quadratic term.

The exponent means that this term is to the second degree.

(Slide #3)

$$ax^2 + bx + c$$

The bx is the linear term. Because it has no exponent, it is to the first degree.

The constant is represented by c. It does not have a variable with it.

(Slide #4)

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

Remember FOIL.

The product of two binomials =

- (the product of the First terms)
- + (the product of the Outside terms)
- + (the product of the Inside terms)
- + (the product of the Last terms)

$$(x + 5)(x + 2)$$

= $(x)(x) + 2x + 5x + 10$
= $x^2 + 7x + 10$

(Slide #8)

$$4x^2 - 11x + 6$$

In the previous examples, a (the coefficient of χ^2) has been equal to 1. In the example above, it is 4. That is accounted for by adding the factors of a and c.

(Slide #17)

$$ax^2 + bx - c$$

If the last term has a negative or minus sign in front of it, you must subtract the factors of a and c to find the correct middle term. The signs in the two binomials will be different.

(Slide #25)

$$x^2 - 25$$

Look at this quadratic. The middle term is missing because "b" = 0.

To factor the quadratic, we must find factors of c that are equal. If we do, the inside and outside products will cancel out. This means that c must be a square and that the second term of each binomial will be the square root of c.

(Slide #29)

Khan Academy helpful video:

http://www.khanacademy.org/math/trigonometry/polynomial_and_rational/quad_factoring/v/factoring-quadratic-expressions

You should be able to solve the following problem prior to moving to the Practice Test:

What are the factors of this quadratic trinomial? $3x^2-7x+4$ Type the correct answer, then press Enter.

Polynomial Equations (L 038) ***Story problems based on quadratics***

An 8 ft. by 10 ft. rectangular garden has a cement walk of uniform width surrounding the garden. If the total area of the garden and the walk is 120 sq. ft., what is the width of the cement walk?



(Slide #14)

We are trying to find the width of the walk.

Let x = width of the walk

2x + 8 = width of walk and garden

2x + 10 = length of walk and garden

$$(2x + 8)(2x + 10) = 120$$

 $4x^2 + 36x + 80 = 120$

(Slide #15)

$$4x^2 + 36x + 80 = 120$$

$$4x^2 + 36x - 40 = 0$$

$$4(x^2 + 9x - 10) = 0$$

$$4(x + 10)(x - 1) = 0$$

Width cannot be negative.

$$x - 1 = 0$$

 $x = 1$

(Slide #16)

The cement walk is one foot wide.



$$8 + 1 + 1 = 10$$
 ft. = total width $10 + 1 + 1 = 12$ ft. = total length $10(12) = 120$ sq. ft. = total area

(Slide #17)

A rectangular garden has a perimeter of 48 cm and an area of 140 sq. cm. What is the length of this garden?

14 cm 14 sq cm 12 in 12 cm

Click on the correct answer.

Negative Exponents (L 039)

EXPONENT REVIEW

$$\mathbf{a}^2 \cdot \mathbf{a}^5 = \mathbf{a}^7$$
 $(\mathbf{a}^4)^6 = \mathbf{a}^{24}$
 $(\mathbf{a}^3\mathbf{b}^7)^6 = \mathbf{a}^{18}\mathbf{b}^{42}$

$$\frac{\mathbf{a}^7}{\mathbf{a}^5} = \mathbf{a}^2$$

$$\frac{\mathbf{a}^5}{\mathbf{a}^7} = \frac{\mathbf{1}}{\mathbf{a}^2}$$

(Slide #1)

NEGATIVE EXPONENTS

Negative exponents are a shortcut for representing fractions with exponents.

A negative exponent represents 1 over the base raised to the opposite of the exponent.

$$\mathbf{a}^{-2} = \frac{1}{\mathbf{a}^2}$$

(Slide #4)

Choose the answer that uses positive exponents to correctly simplify this expression.

$$\frac{6a^{-4}b^{-3}}{9a^2b^{-2}}$$

 $\frac{A}{\frac{1}{3a^{6}b}}$

B 2

C 2 3a⁶b

Type the letter that represents the correct answer, then press Enter.

		П

Scientific Notation (L 040)

A significant digit is any nonzero digit, or any zero, which serves a purpose other than to locate a decimal point.

(Slide #7)

Look at these examples of significant digits.

0.0234 - 3 significant digits

34.51 - 4 significant digits

70,908 - 5 significant digits

(Slide #8)

$$25,000 =$$

 $2.5 \times 10^4 - 2$ significant digits

 $2.50 \times 10^4 - 3$ significant digits

2.500 × 10⁴ - 4 significant digits

 $2.5000 \times 10^4 - 5$ significant digits (and the most accurate).

(Slide #10)

Khan Academy helpful video:

http://www.khanacademy.org/math/arithmetic/exponents-radicals/scientific-notation/v/scientific-notation-3--new

Choose the correct scientific notation for this number.

.0000567

 5.67×10^{-5}

 5.67×10^{-4}

 5.67×10^{5}

Click on the correct answer.

Rational Operations 1 (L 041)

ADDING RATIONAL EXPRESSIONS

A rational expression is simply a fraction with some unknowns in it.

To add or subtract a rational expression, as in fractions, a common denominator must be found before operations can be performed.

(Slide #1)

The same process can be used with rational expressions.

$$\frac{3}{a^2b} + \frac{5}{ab^2}$$

The common denominator is a²b².

(Slide #4)

Now get a piece of paper and simplify this problem.

Test yourself by completing the problem before you go further. Then compare your work to the steps shown on the following screens.



$$\frac{a^2}{a-3} - \frac{4}{5a-15}$$

(Slide #23)

$$\frac{2}{x} + \frac{4}{x^2 - 9} - \frac{6}{x^2 - 3x}$$

The common denominator is x(x - 3)(x + 3).

(Slide #33)

You should be able to solve the following problem prior to moving to the Practice Test:

Click on the least common denominator for these expressions.

$$\frac{3x}{x^2 + x - 2} \qquad \frac{5x}{x^2 - 4x + 4}$$

$$(x-2)(x+1)(x+2)^{2}$$

$$(x-2)(x-1)(x-2)^{3}$$

$$(x+2)(x-1)(x-2)^{2}$$

$$(x-2)(x-1)(x-2)^3$$

$$(x + 2)(x - 1)(x - 2)^2$$

Rational Operations 2 (L 042)

*Dividing rational polynomials is easy. Simply flip the ratio to make the division problem a multiplication problem. Once you have done this, this particular lesson is really no different than the previous lesson.

Example:

$$\frac{(3x-6)}{(2x-8)} \div \frac{(2x-4)}{(x-4)}$$

Flip over the rational expression on the right and multiply.

$$\frac{(3x-6)}{(2x-8)} \cdot \frac{(x-4)}{(2x-4)}$$

(Slide #16)

On your paper, factor, simplify, and find the answer. Remember to flip the rational expression that follows the division sign.

$$\frac{x^3 - 49x}{x^2 - 8x + 7} \cdot \frac{2x^2 - 2}{4x^2} \div \frac{7x + 7}{x(x + 7)}$$

(Slide #18)

You should be able to solve the following problem prior to moving to the Practice Test:

Solve this problem.

$$\frac{x^2 - 25}{(x + 5)^2} \cdot \frac{2x - 10}{4x - 20}$$

A B C D
$$\frac{x-5}{2(x-5)} \qquad \frac{2(x-5)}{x+5} \qquad \frac{x+5}{2(x-5)} \qquad \frac{x-5}{2(x+5)}$$

Type the letter that represents the correct answer, then press Enter.



Simplifying Rationals (L 043)

GCF

$$2x^2 + 4x = 2x(x + 2)$$

Quadratic Trinomial

$$4x^2 + 11x + 6 = (4x + 3)(x + 2)$$

Perfect Square

$$25a^2 - 60ab + 36b^2 = (5a - 6b)^2$$

Difference of Two Squares

$$9x^2 - 25 = (3x - 5)(3x + 5)$$

(Slide #6)

Let's simplify this expression.

$$\frac{4-2x}{x^2-4}$$

The x in the numerator is negative, but in the denominator it is positive. This can be changed so that it results in a possible common factor.

(Slide #9)

SOLVING RATIONAL EQUATIONS

Rational equations can be solved by using the zero-product property.

(Slide #18)

Example:

$$a^{2} - 6a = -5$$
 $a^{2} - 6a + 5 = 0$
 $(a - 1)(a - 5) = 0$
 $a - 1 = 0$
 $a - 5 = 0$
 $a = 1$ or $a = 5$

(Slide #19)

SOLVE AND CHECK THIS PROBLEM.

$$\frac{7x-6}{x^2+x-6}-\frac{x}{x+3}=\frac{2}{x-2}$$

The common denominator is (x + 3)(x - 2).

Multiply each rational term by the common denominator

$$(x + 3)(x - 2)$$
.

(Slide #26)

Khan Academy helpful video:

http://www.khanacademy.org/math/trigonometry/polynomial_and_rational/simplifying-rational-expressions/v/simplifying-rational-expressions-3

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify.

$$\frac{x^2 - 5x + 6}{x^2 - 9}$$

 $\frac{x+2}{x+3}$

 $\frac{x-2}{x+3}$

 $\frac{x-2}{x-3}$

Click on the correct answer.

Complex Rationals (L 044)

 $\frac{1}{x}$

<u>a</u>b/2

A complex rational expression is simply an expression that has a rational expression in its numerator and/or its denominator.

(Slide #2)

Find the denominator by multiplying the two "middle parts" of this problem (x and 3).

Perform this operation and put the product in the denominator's position.

 $\begin{bmatrix} \frac{2}{X} \\ \frac{3}{X} \end{bmatrix}$

Now find the numerator by multiplying the two outside factors.

This product will become the numerator.

$$\frac{2x}{3x} = \frac{2}{3}$$

(Slide #7)

Some things to remember:

When adding or subtracting fractions, you must find a common denominator. To simplify means to REDUCE!

When multiplying or dividing fractions, if common factors are in both numerator and denominator, 1 will simplify the fraction.

(Slide #43)

You should be able to solve the following problems prior to moving to the Practice Test:

Click on the answer that correctly simplifies this expression.

$$\frac{\frac{3}{x}}{2 + \frac{3}{2x}}$$

$$\frac{4}{6x+3}$$

$$\frac{6}{4x+3}$$

$$\frac{x}{4+3y}$$

Click on the answer that correctly simplifies this expression.

$$\frac{3}{x^2 - 16}$$

$$\frac{2x}{x + 4}$$

$$\frac{3}{2x(x-4)}$$

$$\frac{3x^2}{x+4}$$

$$\frac{3x}{2x-4}$$