Roots and Radicals (L 001)

Roots and Radicals

The product of two equal numbers or expressions is called a perfect square.

$x^4x^4 = x^8$ $\quad\quad$ $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

$4 \cdot 4 = 16$

$.09 \cdot .09 = .0081$ $\quad\quad$ $(x^6y^8)(x^6y^8) = x^{12}y^{16}$

(Slide #1)

Similarly, the product of three equal numbers or expressions is called a perfect cube.

$x^4x^4x^4 = x^{12}$ $\quad\quad$ $4 \cdot 4 \cdot 4 = 64$

$\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64}$ $\quad\quad$ $(x^6x^8)(x^6y^8)(x^6y^8) = x^{18}y^{24}$

(Slide #3)

Roots are indicated by a radical sign, $\sqrt{}$.

The index, n, indicates which root to take, such as the square root or the cube root, etc.

Finally, the radicand is the number or expression for which we wish to find this root.

Note that if no index appears, a square root is implied.

(Slide #7)

THE REAL nth Roots of a.
$\sqrt[n]{a}$ or $-\sqrt[n]{a}$

	a > 0	a < 0	a = 0
n even	one positive root one negative root	no real roots	one real root, 0
n odd	one positive root no negative roots	no positive roots one negative root	

Remember, the principal root is the positive root, unless there is no positive root.

(Slide #16)

Reminder: When you find the nth root of an even power (2nd, 4th, 6th, ...), and the result is an odd power (1st, 3rd, 5th, ...), you must take the absolute value of the result to make certain the value is nonnegative. *(Slide #24)*

Khan Academy helpful video: http://www.khanacademy.org/math/arithmetic/exponents-radicals/radical-radicals/v/simplifying-radicals

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify.

$$\sqrt[3]{27x^9}$$

$3x^9$	$3x^6$
$3x^3$	$9x^3$

Real Number Properties 1 (L 002)

Product Property of Radicals

For any real numbers "a" and "b," and any integer "n," where n > 1,

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

(Slide #5)

Two binomials of the form a + b, a - b are called **"conjugates."**

For instance, the conjugate of $2 + \sqrt{3}$ is $2 - \sqrt{3}$.

(Slide #14)

Quotient Property of Radicals

For any real numbers "a" and "b" with "b" $\neq$ 0 and any integer "n" where n > 1,

$$\sqrt[n]{\frac{a^n}{b^n}} = \frac{\sqrt[n]{a^n}}{\sqrt[n]{b^n}}$$

(Slide #17)

An nth root is in simplest radical form when:

1. The radicand contains no perfect nth power.
2. The radicand contains no fractions.

and

3. The denominator contains no radicals.

(Slide #19)

Rationalize:

$$\frac{\sqrt{8}}{\sqrt{3}-\sqrt{2}} = \frac{(2\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{2\sqrt{6}+4}{3-2}$$

$$= 2\sqrt{6}+4$$

To rationalize a binomial with radicals, multiply by a form of one using the conjugate.

(Slide #29)

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify.

$$\frac{\sqrt{50}}{\sqrt{2}}$$

Real Number Properties 2 (L 003)

With monomials, we have combined like terms:

$$2x + 3y - x + 4y = x + 7y$$

With radicals, we will combine like radicals:

$$2x\sqrt{5} + 3y\sqrt{7} - x\sqrt{5} + 4y\sqrt{7} = x\sqrt{5} + 7y\sqrt{7}$$

(Slide #3)

Use these rules for simplifying sum or difference of radicals.

*Simplify each radical.

*Combine like radical terms using the distributive property.

(Slide #9)

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify.

$$8\sqrt{45} - \sqrt{80}$$

$28\sqrt{5}$	$7\sqrt{35}$
$7\sqrt{5}$	$20\sqrt{5}$

Rational Exponents (L 004)

Let's begin by defining a rational exponent.

If "a" and "n" are positive integers, and $\sqrt[n]{b}$ is a real number, then

$$b^{\frac{1}{n}} = \sqrt[n]{b}$$

and

$$b^{\frac{a}{n}} = \sqrt[n]{b^a},$$

or

$$b^{\frac{a}{n}} = (\sqrt[n]{b})^a$$

(Slide #3)

This means that the following are true:

$$\sqrt{4} = 4^{\frac{1}{2}}$$

$$\sqrt[3]{8} = 8^{\frac{1}{3}}$$

$$\sqrt[3]{27^2} = 27^{\frac{2}{3}}$$

(Slide #4)

Khan Academy helpful videos: http://www.khanacademy.org/math/algebra/exponent-equations/exponent-properties-algebra/v/rational-exponents-and-exponent-laws

AND

http://www.khanacademy.org/math/algebra/exponent-equations/exponent-properties-algebra/v/more-rational-exponents-and-exponent-laws

You should be able to solve the following problem prior to moving to the Practice Test:

Choose the answer that correctly shows the exponential form of this expression.

$$\sqrt[3]{3x^2y^4}$$

$3x^{\frac{2}{3}}y^{\frac{4}{3}}$	$3x^{\frac{1}{3}}y$
$(3xy)^{\frac{2}{3}}$	$3^{\frac{1}{3}}x^{\frac{2}{3}}y^{\frac{4}{3}}$

<u>Equations (L 005)</u>

Step 1

Isolate one radical.

In this example, the radical is isolated on one side by itself.

$$\sqrt{x+2} = 5$$

(Slide #5)

Step 2

Square both sides of the equation to eliminate the radical.

$$(\sqrt{x+2})^2 = (5)^2$$

$$x + 2 = 25$$

(Slide #6)

Step 3

Solve for the variable.

$$x + 2 = 25$$
$$x = 23$$

(Slide #7)

Check the solution in the original equation.

(This can often be done mentally.)

$$\sqrt{x+2} = 5$$

$$\sqrt{23+2} = 5$$

$\sqrt{25}$ does equal 5, so 23 is the answer.

$$(\sqrt{x+2})(\sqrt{x+2}) = x+2$$

(Slide #8)

Khan Academy helpful video: http://www.khanacademy.org/math/algebra/exponent-equations/radical_equations/v/solving-radical-equations

You should be able to solve the following problems prior to moving to the Practice Test:

Solve this radical equation.

$$\sqrt{1-3x} = -7$$

x =

Type the correct answer, then press Enter.
Type NO REAL SOLUTION for answers that fit this description.
There may be more than one answer.

Solve this radical equation.

$$3\sqrt{x} + 3 = 15$$

x =

Type the correct answer, then press Enter.
Type NO REAL SOLUTION for answers that fit this description.
There may be more than one answer.

Imaginary Numbers (L 006)

Note that: $i = \sqrt{-1} = i$

$i^2 = (\sqrt{-1})(\sqrt{-1}) = -1$

Powers of *i* follow an interesting pattern.

$i^3 = (i^2)(i) = (-1)(i) = -i$

$i^4 = (i^2)(i^2) = (-1)(-1) = 1$

$i^5 = (i^4)(i) = (1)(i) = i$

$i^6 = (i^4)(i^2) = (1)(-1) = -1$

etc.

Do you see the pattern?

(Slide #8)

Khan Academy helpful video:
http://www.khanacademy.org/math/trigonometry/imaginary_complex_precalc/i_precalc/v/introduction-to-i-and-imaginary-numbers

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify:

i^{54}

$-i$	-1
i	1

Complex Numbers 1 (L 007)

Addition and Subtraction of Complex Numbers

A number written in the form a + b*i*,
where a and b are real numbers and *i* is the imaginary number,
is called a complex number.

(Slide #1)

If a and b are real numbers, then

$$\|a + bi\| = \sqrt{a^2 + b^2}.$$

$\|a + bi\|$ is called the MAGNITUDE of the complex number a + b*i*. *(Slide #6)*

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify:

$$(11 + 2\sqrt{-50}) + (-12 + 3\sqrt{-72})$$

$23 + 16i\sqrt{2}$	$-1 - 8i\sqrt{2}$
$-1 + 28i\sqrt{2}$	$23 + 28i\sqrt{2}$

Complex Numbers 2 (L 008)

The same principle also holds for complex conjugates, a + b*i* and a - b*i*.

The product of these two numbers will eliminate the imaginary term!

$$(a - bi)(a + bi) = a^2 + b^2$$

(Slide #16)

$$\frac{2 + 5i}{5 - 7i}$$

$$\frac{5 - 7i}{2 + 5i} = \frac{(5 - 7i)(2 - 5i)}{(2 + 5i)(2 - 5i)}$$

$$= \frac{10 - 25i - 14i + 35i^2}{4 - 25i^2}$$

$$= \frac{-25 - 39i}{29}$$

(Slide #28)

Khan Academy helpful video: http://www.khanacademy.org/test-prep/california-standards-test/california-standards-test-algebra-2/v/algebra-ii--complex-numbers-and-conjugates

You should be able to solve the following problem prior to moving to the Practice Test:

Simplify to a + b*i* form:

$$(5 + 3i)(2 - 3i)$$

Quadratic Equations 1 (L 009)

Many equations, however, do not factor.

For example,

$$x^2 + 8x - 1 = 0.$$

Equations of this type require a different method of solving.

(Slide #3)

$$x^2 + 8x - 1 = 0$$

$$x^2 + 8x + 16 = 17$$

$$(x + 4)^2 = 17$$

$$\sqrt{(x + 4)^2} = \pm\sqrt{17}$$

$$x + 4 = \pm\sqrt{17}$$

$$x = -4 \pm \sqrt{17}$$

(Slide #10)

One rule that has not been mentioned for completing the square is that the leading coefficient must be 1.

If the leading coefficient is not 1, we must divide to make it so.

(Slide #13)

Let's sum up the steps for completing the square for $ax^2 + bx + c = 0$.

(Slide #22)

First, move c to the right side and divide all terms by a:

$$x^2 + \frac{(bx)}{a} = \frac{-c}{a}$$

Then take half of $\frac{b}{a}$, square it, and add it to both sides:

$$x^2 + \left(\frac{b}{a}\right)x + \frac{b^2}{(2a)^2} = \frac{-c}{a} + \frac{b^2}{(2a)^2}$$

Now factor the left, take the square root of both sides, and solve!

(Slide #23)

Khan Academy helpful video:
http://www.khanacademy.org/math/trigonometry/polynomial_and_rational/quad_formula_tutorial/v/solving-quadratic-equations-by-completing-the-square

You should be able to solve the following problem prior to moving to the Practice Test:

Choose the correct value for x when this equation is solved by completing the square.

$$x^2 - 12x + 4 = 0$$

$4 \pm 6\sqrt{2}$	$2 \pm 4\sqrt{2}$	$6 \pm 4\sqrt{2}$

Quadratic Equations 2 (L 010)

This, then, is the Quadratic Formula!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now to solve a quadratic equation, all we need to know are a, b, and c from

$ax^2 + bx + c = 0$!

(Slide #7)

Khan Academy helpful video:

http://www.khanacademy.org/math/algebra/quadratics/quadratic_odds_ends/v/introduction-to-the-quadratic-equation

You should be able to solve the following problem prior to moving to the Practice Test:

Use the quadratic formula to choose the correct solution to this equation.

$$2x^2 - 10x + 5 = 0$$

$\frac{3 \pm \sqrt{15}}{2}$

$\frac{5 \pm \sqrt{15}}{2}$

The Discriminant (L 011)

The quantity $b^2 - 4ac$ is called the discriminant.

(Slide #2)

The discriminant can be used to determine the number of times the graph of a quadratic equation crosses the x-axis.

(Slide #4)

Let's summarize!

If $b^2 - 4ac < 0$, the solution is two complex conjugates, and the graph does not cross the x-axis.

(Slide #17)

If $b^2 - 4ac > 0$ and is NOT a perfect square, the solution is two real, irrational conjugates, and the graph crosses the x-axis twice.

(Slide #18)

If $b^2 - 4ac > 0$ and is a perfect square, the solution is two real, rational, unequal numbers, and the graph crosses the x-axis twice.

(Slide #19)

If $b^2 - 4ac = 0$, the solution is one real, rational number, and the graph is tangent to the x-axis.

(Slide #20)

Khan Academy helpful video:
http://www.khanacademy.org/math/trigonometry/polynomial_and_rational/quad_formula_tutorial/v/discriminant-of-quadratic-equations

You should be able to solve the following problem prior to moving to the Practice Test:

Using the discriminant, determine the number and type of solutions for this equation.

$$3x^2 + 4x + 2 = 0$$

- two real, rational, unequal numbers
- one real, rational number (double)
- two real, irrational, unequal numbers
- two complex conjugate numbers

Roots (L 012)

In other words, the sum of the roots of a quadratic equation equals $-\frac{b}{a}$, and the product equals $\frac{c}{a}$.

(Slide #5)

You should be able to solve the following problem prior to moving to the Practice Test:

Choose the quadratic equation that has a leading coefficient of 1 and solutions 3 and -2.

$x^2 + x + 5 = 0$	$x^2 - x - 5 = 0$
$x^2 + x + 6 = 0$	$x^2 - x - 6 = 0$

Quadratic Equations 3 (L 013)

This is probably the most difficult lesson in Algebra II. Make sure you understand the study portion completely prior to moving on.

Can $x - 4\sqrt{x} - 5 = 0$ be put in quadratic form?

$$(\sqrt{x})^2 - 4\sqrt{x} - 5 = 0$$
$$(\sqrt{x} - 5)(\sqrt{x} + 1) = 0$$
$$\sqrt{x} = 5, -1$$

Yes.

(Slide #6)

You should be noticing a correlation between the degree of the equation and the number of solutions it has.

(Slide #8)

$$\frac{2}{x-1} + \frac{1}{x+1} = 3$$

To solve this problem, multiply both sides by the common denominator (x - 1)(x + 1) to get rid of the fraction:

$$\frac{2(x+1)(x-1)}{x-1} + \frac{1(x+1)(x-1)}{x+1} = 3(x + 1)(x - 1)$$

$$2(x + 1) + 1(x - 1) = 3(x + 1)(x - 1)$$
$$2x + 2 + x - 1 = 3x^2 - 3$$
$$3x^2 - 3x - 4 = 0$$

Solving with the quadratic formula yields this answer:

$$x = \frac{3 \pm \sqrt{57}}{6}$$

(Slide #14)

You should be able to solve the following problem prior to moving to the Practice Test:

Solve this equation by using the quadratic form.

$$x^4 - 40x^2 + 144 = 0$$

x = [ ]

Problem Solving (L 014)

When solving quadratic equations, there are usually two solutions to consider.

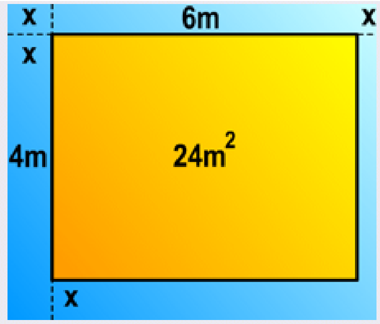
Be sure the solution you choose makes sense!

(Slide #2)

To find the area of a rectangle, multiply the length by the width.

That makes an equation!

$$(2x + 4)(2x + 6) = 48$$



We suggest that you get a pencil and paper and work along as this problem is solved to make sure that you understand it.

(Slide #6)

Use these steps for problem solving:

Draw an accurate picture.

Gather information given and decide what is to be found.

Check your solutions and choose the one that makes sense.

(Slide #20)

You should be able to solve the following problem prior to moving to the Practice Test:

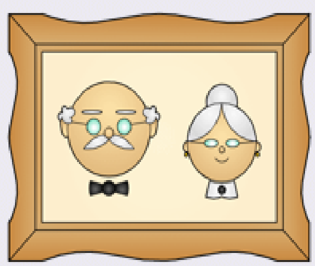
If a 25 in. by 32 in. picture has a wooden frame "x" in. wide surrounding it, which equation can be used to find the total area of the picture and frame?

$$A = 800 - 114x + 4x^2$$

$$A = 2x^2 + 114x + 800$$

$$A = 4x^2 + 114x + 800$$

$$A = x^2 + 114x + 800$$



Quadratic Relations (L 015)

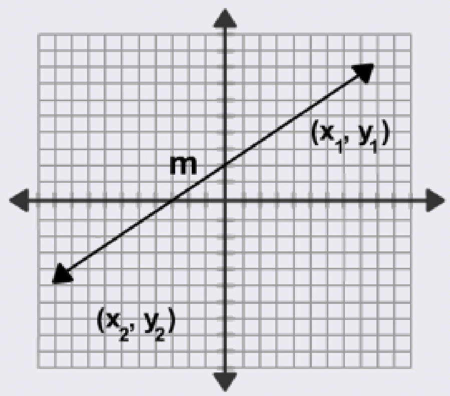
DISTANCE FORMULA

This means that the distance "d" between two points A and B in a coordinate plane can be found by this formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Slide #9)

A line segment with endpoints at (x_1, y_1) and (x_2, y_2), will have a midpoint "M" at the point $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



(Slide #16)

You should be able to solve the following problem prior to moving to the Practice Test:

Find the distance from Point A (2, -3) to Point B (2, 4).

______ units

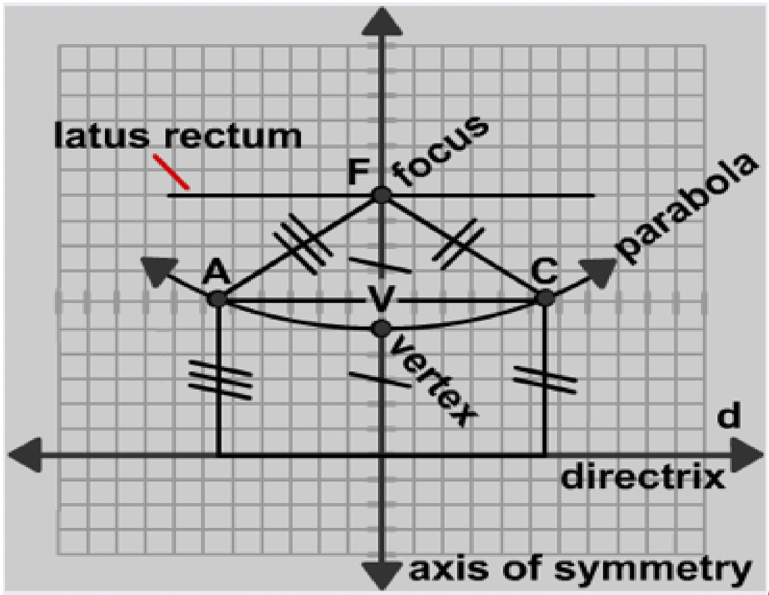
Parabolas (L 016)

Characteristics of a Parabola

A parabola is the set of points in a plane equidistant from a fixed point (focus) and a fixed line (directrix).

Neither the focus nor the directrix lies on the parabola.

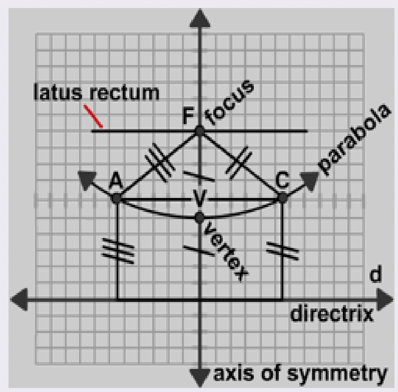
(Slide #1)



(Slide #2)

The parabola is symmetrical about its axis, which is the line through the focus perpendicular to the directrix.

The line segment passing through the focus and perpendicular to the axis is called the latus rectum.



(Slide #4)

Standard form (given x^2):
$$y = a(x-h)^2 + k, \quad a \neq 0$$

Standard form (given y^2):
$$x = a(y-k)^2 + h, \quad a \neq 0$$

(Slide #9)

Let's begin with a. If a is positive, the parabola will open upward or to the right (depending on whether we have x^2 or y^2).

If a is negative, the parabola will open downward or to the left.

(Slide #11)

SUMMARY CHART

Standard Form	$y = a(x-h)^2 + k$	$x = a(y-k)^2 + h$
Vertex	(h, k)	(h, k)
Axis of symmetry	$x = h$	$y = k$
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Opening	Upward if $a > 0$ Downward if $a < 0$	Right if $a > 0$ Left if $a < 0$

(Slide #30)

Khan Academy helpful video: http://www.khanacademy.org/math/algebra/conic-sections/conic_parabolas/v/parabola-focus-and-directrix-1

You should be able to solve the following problem prior to moving to the Practice Test:

What does the point (3, 4) represent in $y = 5(x - 3)^2 + 4$?

- focus
- directrix
- vertex
- axis of symmetry

Graphing Parabola (L_017)

Notice that the parabola will be wide when a is fractional.

(Slide #6)

To determine where to shade, select a point that is NOT on the parabola to plug in. (0, 0) would be an easy choice.

(Slide #24)

You should be able to solve the following problem prior to moving to the Practice Test:

Choose the correct description of the graph of this parabola.

$$y \le (x-3)^2 + 2$$

- solid line shaded inside parabola
- dotted line shaded inside parabola
- dotted line shaded outside parabola
- solid line shaded outside parabola

Circles (L 018)

The standard form for the equation of a circle is

$$(x-h)^2 + (y-k)^2 = r^2,$$

where (h, k) is the center and r is the radius of the circle.

(Slide #3)

Consider the circle $x^2 + y^2 + 12x - 8y - 12 = 0$.

We need this equation in standard form, so we'll need to complete the square for x^2 and y^2.

$$(x^2 + 12x) + (y^2 - 8y) = 12$$

$$(x^2 + 12x + 36) + (y^2 - 8y + 16) = 12 + 36 + 16$$

$$(x+6)^2 + (y-4)^2 = 64$$

Now it looks like the equation of a circle with center (-6, 4) and radius 8!

(Slide #23)

Make sure that you are able to tell the difference between the equation of a parabola and that of a circle, even if they aren't in standard form.

As you noticed with the parabola, either the x was squared or the y was squared, but NOT BOTH.

(Slide #24)

On the circle, BOTH x and y are squared and ADDED together.

(Slide #25)

Khan Academy helpful video:
http://www.khanacademy.org/math/trigonometry/conics_precalc/circles-tutorial-precalc/v/conic-sections--intro-to-circles

You should be able to solve the following problem prior to moving to the Practice Test:

Find the center and radius of the circle with this equation.

$$x^2 + (y - 4)^2 = 64$$

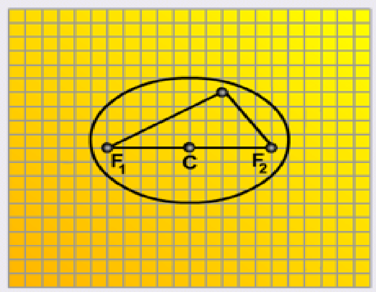
(0, 4), 8	(0, -4), 64
(0, 4), 64	(0, -4), 8

Ellipses (L 019)

Ellipse Characteristics

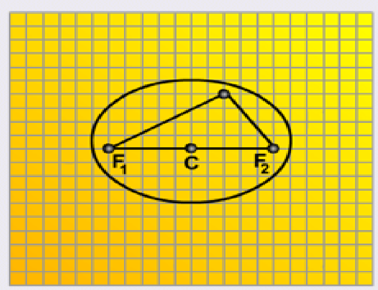
An ellipse is the set of all points, "P," for which the sum of the distances from "P" to two fixed points is a constant.

These fixed points are called foci (plural for focus).



(Slide #1)

The distances from the foci to point P on the curve are called focal radii. The midpoint C between the two foci is the center of the ellipse.

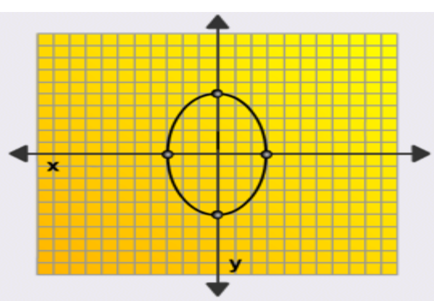


(Slide #2)

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

Now a is the length from the center to one end of the major axis (a is always the largest), b is the length from the center to one end of the minor axis, and c is the length from the center to one focus.

The relationship between these is $c^2 = a^2 - b^2$.



(Slide #11)

The standard form of the equation of an ellipse with center (h, k):

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \text{ (major axis parallel to x-axis)}$$

or

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \text{ (major axis parallel to y-axis)}$$

(Slide #13)

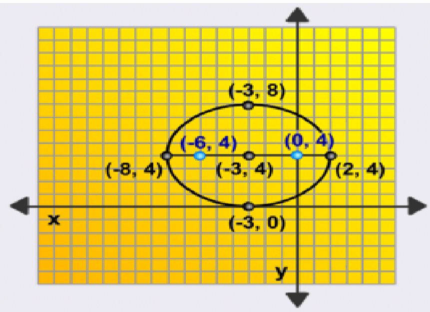
$$\frac{(x+3)^2}{25} + \frac{(y-4)^2}{16} = 1$$

Plot the center at (-3, 4).

Because the major axis is horizontal, go left and right 5 units from the center.

For the minor axis, go up and down 4 units from the center.

Using the endpoints of the axes, sketch the graph of the ellipse.



(Slide #17)

Summary

Equation in Standard Form	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Center	(h, k)	(h, k)
Major axis	parallel to x-axis	parallel to y-axis
Minor axis	parallel to y-axis	parallel to x-axis
Foci	$c^2 = a^2 - b^2$	$c^2 = a^2 - b^2$

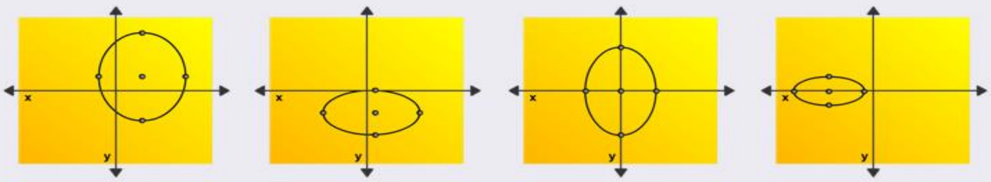
(Slide #25)

Khan Academy helpful video: http://www.khanacademy.org/math/algebra/conic-sections/conic_ellipses/v/conic-sections--intro-to-ellipses

You should be able to solve the following problem prior to moving to the Practice Test:

Choose the correct graph of this ellipse.

$$\frac{(x+4)^2}{4} + y^2 = 1$$

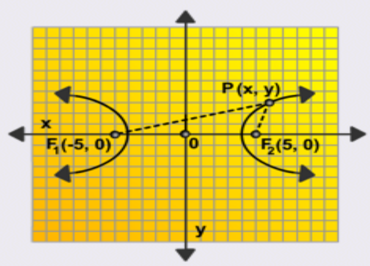


<u>Hyperbola (L 020)</u>

Hyperbola Characteristics

A hyperbola is the locus of all points P in a plane for which the difference of the absolute value of the distances from two fixed points is a constant. These two fixed points are the foci.

This definition is very similar to that of an ellipse, except that with hyperbolas, we are interested in the difference rather than the sum!



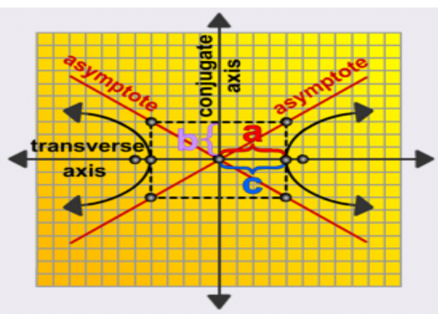
(Slide #1)

A hyperbola has two branches and two axes of symmetry.

The axis that passes through the foci is called the transverse axis.

The conjugate axis passes through the center (which is between the foci) and is perpendicular to the transverse axis.

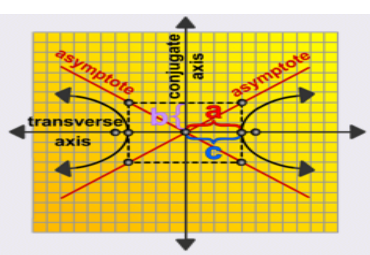
The point on each curve of the hyperbola nearest the center is called a vertex.



(Slide #7)

Similar to the ellipse, the distance from the center to a vertex is "a" units ("a" will always be on the transverse axis), and the distance from the center to a focus is "c" units.

B is measured from the center along the conjugate axis and is useful when working with asymptotes.

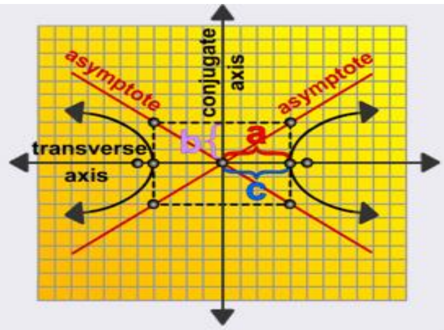


The relationship between the three is

$c^2 = a^2 + b^2$.

(Slide #8)

The asymptotes are lines that the hyperbola curves approach but never touch. They pass through the center of the hyperbola, forming the diagonals of the rectangle that has sides 2a and 2b units.



If the transverse axis is horizontal, the slopes for the asymptotes are $\pm \frac{b}{a}$.

If the transverse axis is vertical, the slopes are $\pm \frac{a}{b}$.

(Slide #9)

If a hyperbola has a center at (h, k), the equation will change much like it did for the ellipse.

For a horizontal transverse axis, the hyperbola equation is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

For a vertical transverse axis, we have

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Note that a^2 is always in the positive fraction!

(Slide #15)

$$25y^2 - 150y - 4x^2 - 16x = -109$$

First, we need to complete the square:

$$25(y^2 - 6y) - 4(x^2 + 4x) = -109$$

$$25(y^2 - 6y + 9) - 4(x^2 + 4x + 4) = -109 + 225 - 16$$

$$25(y-3)^2 - 4(x+2)^2 = 100$$

Divide both sides by 100 to get standard form:

$$\frac{(y-3)^2}{4} - \frac{(x+2)^2}{25} = 1$$

(Slide #23)

Another type of hyperbola is the rectangular hyperbola. It uses the x-axis and y-axis as its asymptotes and is of the form of xy = k.

(Slide #24)

Hyperbola Summary

Equation in Standard Form	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Transverse axis	parallel to x-axis	parallel to y-axis
Conjugate axis	parallel to y-axis	parallel to x-axis
Foci	$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$
Slopes of Asymptotes	$\pm \frac{b}{a}$	$\pm \frac{a}{b}$

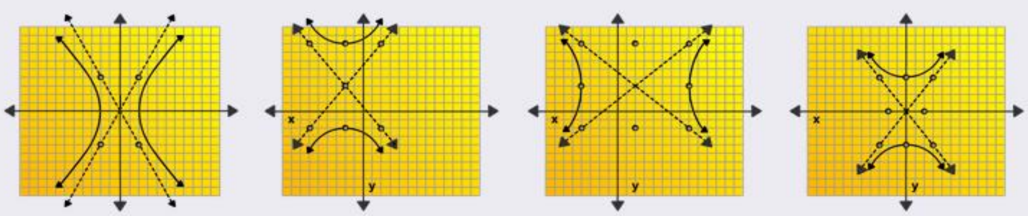
(Slide #32)

Khan Academy helpful video:
http://www.khanacademy.org/math/trigonometry/conics_precalc/hyperbolas-precalc/v/conic-sections--hyperbolas-2

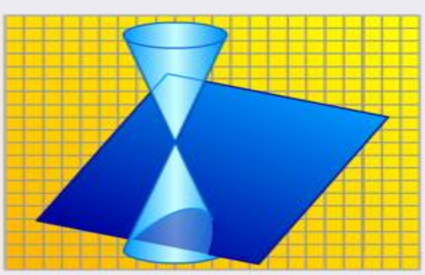
You should be able to solve the following problem prior to moving to the Practice Test:

Choose the graph of this hyperbola.

$$9y^2 - 16x^2 = 144$$



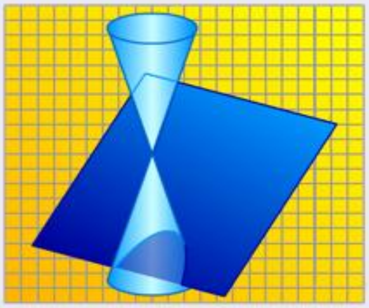
Graphing Relations (L 021)



PARABOLA

$$y = a(x-h)^2 + k$$
$$x = a(y-k)^2 + h$$

(Slide #3)

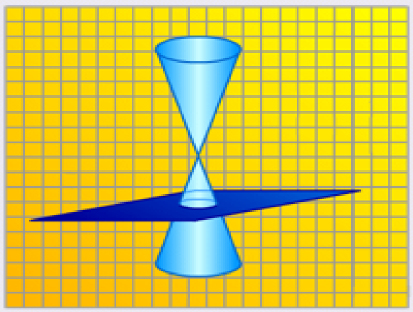


PARABOLA

$y = x^2 + 5$

Either the x or y is squared, but not both.

(Slide #8)

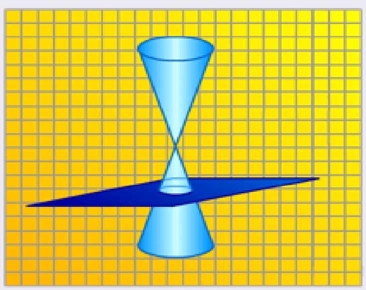


ELLIPSE

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

(Slide #4)

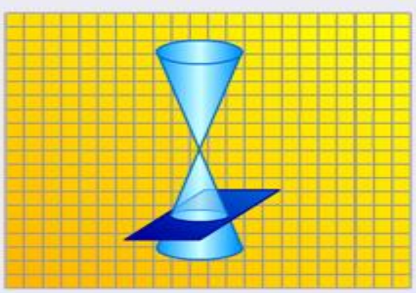


ELLIPSE

$9x^2 + 4y^2 = 36$

Both x and y are squared and added together, but the coefficients are different.

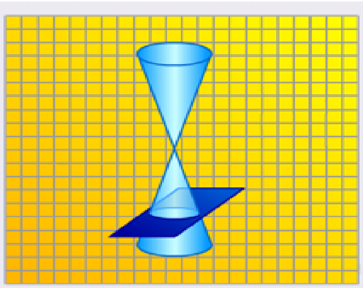
(Slide #9)



CIRCLE

$(x - h)^2 + (y - k)^2 = r^2$

(Slide #5)

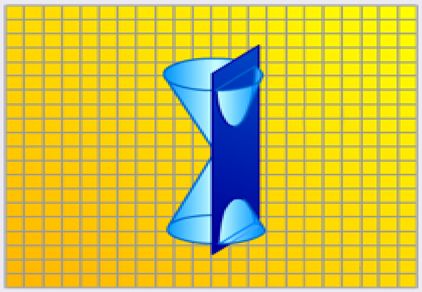


CIRCLE

$x^2 + y^2 = 16$

Both x and y are squared and added together, and the coefficients are the same.

(Slide #10)

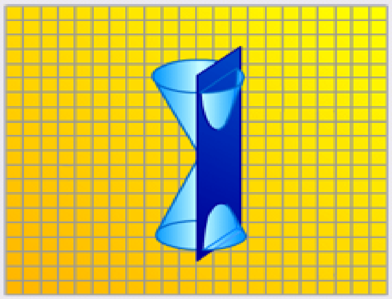


HYPERBOLA

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

(Slide #6)



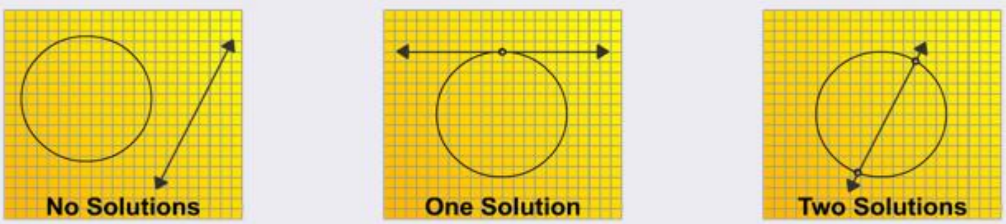
HYPERBOLA

$9x^2 - 4y^2 = 36$

Both x and y are squared, but one is subtracted from the other.

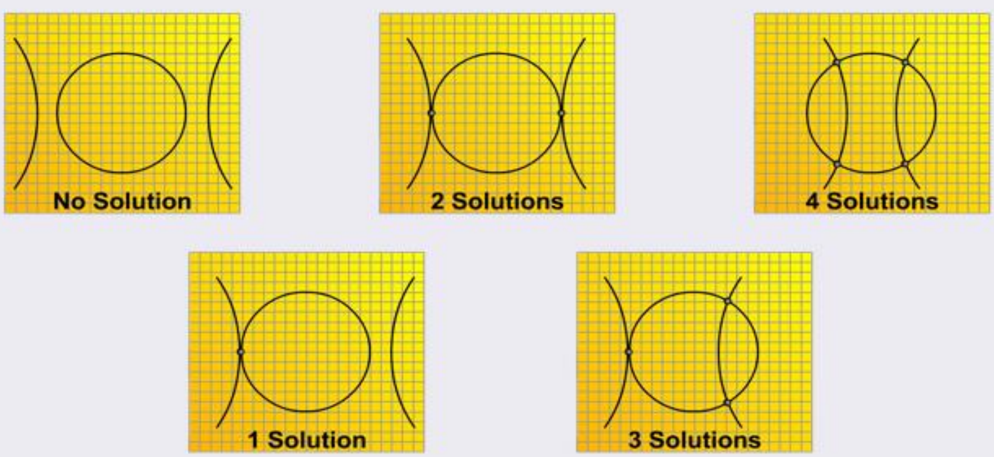
(Slide #11)

In a system of equations comprised of a quadratic and a straight line, the number of intersections could be none, one, or two.



(Slide #16)

The graphs of two quadratic equations could intersect in zero, one, two, three, or four points.



(Slide #17)

Khan Academy helpful video: http://www.khanacademy.org/math/algebra/conic-sections/conics_from_equations/v/algebra-----conic-sections

You should be able to solve the following problem prior to moving to the Practice Test:

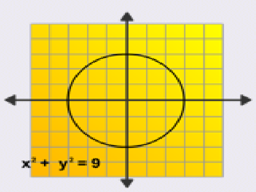
The graph of this equation will be which conic section?

$$4y^2 - x^2 + 10y - 16x = -5$$

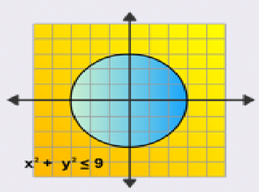
ellipse	circle
parabola	hyperbola

Graphing Inequalities (L 022)

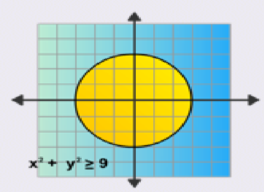
Take one more look at the three graphs and notice how each is shaded.



The graph of $x^2 + y^2 = 9$ is a circle with center at the origin and radius 3.



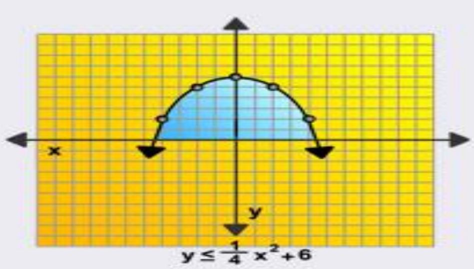
The inequality $x^2 + y^2 \leq 9$ will be the same circle, but shaded inside.

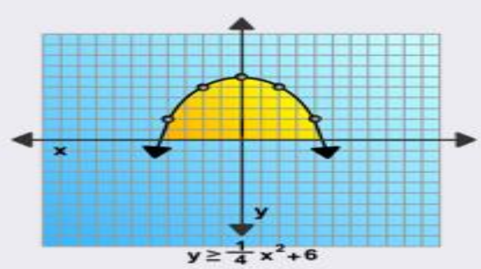


The inequality $x^2 + y^2 \geq 9$ will be shaded outside of the circle.

(Slide #5)

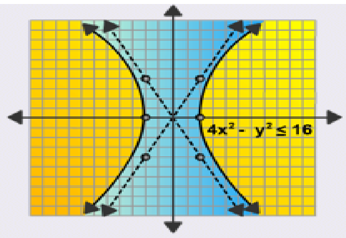
Made into inequalities, $y \leq (-\frac{1}{4})x^2 + 6$ will be shaded inside, and $y \geq (-\frac{1}{4})x^2 + 6$ will be shaded outside.





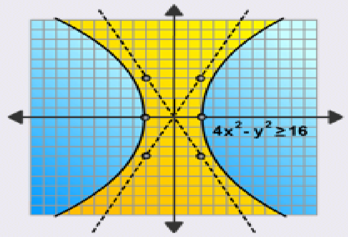
(Slide #11)

For the inequality $4x^2 - y^2 \leq 16$, the portion of the coordinate grid to be shaded (in blue) is between the two branches of the hyperbola.



For the inequality $4x^2 - y^2 \geq 16$, the shaded portion is the opposite of the example above. (Please take note of the blue section of the graph to the right.)

Also remember that the hyperbola itself (the boundary of these regions) is included in both of these cases because of the "or equal to" condition.



(Slide #13)

To determine where to shade an inequality, choose a point not on the graph(s).

If the point makes the inequality statement(s) true, shade the region enclosing that point.

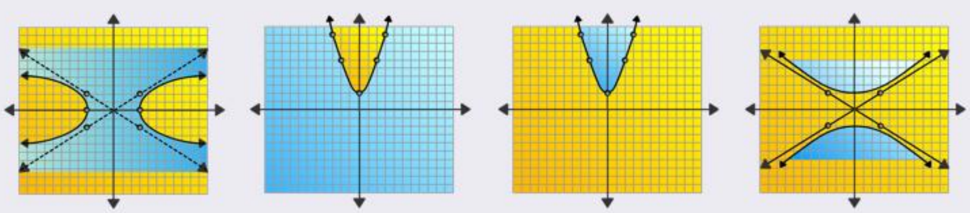
If the point makes the inequality statement(s) false, shade elsewhere. (You may want to check a point in another region.)

(Slide #14)

You should be able to solve the following problem prior to moving to the Practice Test:

Choose the graph of this quadratic inequality.

$$\frac{y^2}{4} - \frac{x^2}{9} \geq 1$$



<u>Variations (L 023)</u>

Here is a summary of the equations of each variation.

direct variation: $y = kx$

inverse variation: $y = \frac{k}{x}$

combined variation: $z = \frac{ky}{x}$

joint variation: $z = kxy$

(Slide #26)

Khan Academy helpful video: http://www.khanacademy.org/math/algebra/algebra-functions/direct_inverse_variation/v/direct-inverse-and-joint-variation

You should be able to solve the following problem prior to moving to the Practice Test:

On a rectangular hyperbola, if one point is (2, 3) and y varies inversely to x, find y when x is 6.

y =

Exponential Functions (L 024)

$$25^{\frac{3}{2}} = (5^2)^{\frac{3}{2}}$$

simplify the fraction $2\left(\frac{3}{2}\right) = 3$,

so $5^3 = 125$

Or take a different approach.

$$25^{\frac{3}{2}} = (25^3)^{\frac{1}{2}} = 15625^{\frac{1}{2}} = 125$$

Which looks easier for you?

(Slide #4)

Remember, a negative exponent must be moved to make it positive.

$$-3x^{-3}y^2$$

$$\frac{-3y^2}{x^3}$$

(Slide #13)

You should be able to solve the following problem prior to moving to the Practice Test:

Choose the exponential form.

$$\sqrt[5]{\sqrt{3}}$$

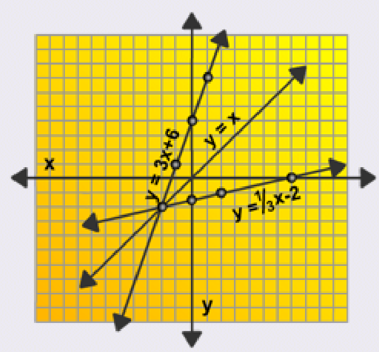
$3^{\frac{5}{2}}$	$3^{\frac{1}{10}}$
$3^{\frac{2}{5}}$	$3^{\frac{1}{7}}$

Inverse Functions (L 025)

Consider the function y = 3x + 6. To find the inverse of this function, we need to interchange the coordinates. We do this by interchanging the x and y and solving again for y:

$$x = 3y + 6$$
$$3y = x - 6$$
$$y = (1/3)x - 2$$

Here are the graphs of the two lines.



(Slide #5)

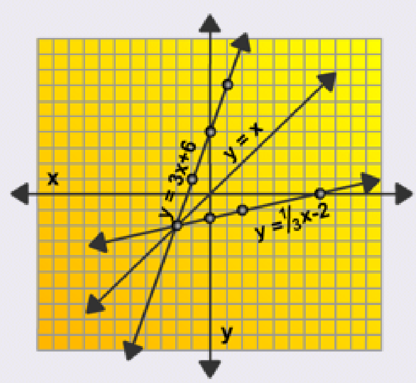
The notation used for inverse functions is

$$f^{-1}$$

We know that

f(x) = 3x + 6 and

$f^{-1} = \left(\frac{1}{3}\right)x - 2$ are inverse functions.



(Slide #12)

Please note that the inverse of a function is not always a function itself.

An example of this is a parabola that opens upward. Its inverse will be a parabola on its side and is therefore not a function.

(Slide #25)

You should be able to solve the following problem prior to moving to the Practice Test:

Choose the inverse function.

$$f^{-1} = \frac{x - 12}{3}$$

$f^{-1} = x + 4$	$f^{-1} = 3(x - 4)$
$f^{-1} = 3x - 12$	$f^{-1} = 3x + 12$

Logarithmic Functions (L 026)

A logarithm (or log, for short) is an exponent. Therefore, any exponential expression can be rewritten in logarithmic form:

For exponential form

$y = b^x$,

the logarithmic form will be

$x = \log_b y$.

(Slide #4)

Properties of Logarithmic Functions

The domain (x-coordinate) is the set of positive real numbers.

The range (y-coordinate) is the set of all real numbers.

The x-intercept of the graph is 1.

The y-axis is an asymptote of the graph.

The function is one-to-one.

(Slide #8)

$\log_2 16 = 4$

What is a logarithm?
It's an exponent.

So, the exponent is 4, with a base of 2.

The exponential form is $2^4 = 16$.

(Slide #12)

Khan Academy helpful video: http://www.khanacademy.org/math/algebra/logarithms-tutorial/logarithm_basics/v/introduction-to-logarithms

You should be able to solve the following problems prior to moving to the Practice Test:

Choose the logarithmic form.

$$8^2 = 64$$

$\log_8 2 = 64$	$\log_8 64 = 2$
$\log_2 64 = 8$	$\log_2 8 = 64$

Solve for x:

$$\log_8 x = 3$$

x = [ ]

<u>Exponential Equations (L 027)</u>

Exponential Equations

Exponential equations are equations in which the variables appear as exponents. These equations can be solved by using logarithmic functions.

(Slide #1)

Some Properties of Logarithms

If x, y, and b ($b \neq 1$) are positive real numbers and a is any real number, then the following are true:

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$\log_b(y^a) = a \cdot \log_b(y)$$

(Slide #2)

To solve an equation such as $2^x = 27$, first take the log of both sides:

$$\log 2^x = \log 27$$

(Slide #4)

$$2^x = 27$$
$$\log 2^x = \log 27$$

Use the third property to "bring the variable down."

$$x \log 2 = \log 27$$

(Slide #5)

$$2^x = 27$$
$$\log 2^x = \log 27$$
$$x \log 2 = \log 27$$

Now just solve for x.

$$x = \frac{\log 27}{\log 2}$$

(Slide #6)

$$3^{x+1} = 80.5.$$

Take the log of both sides and apply property #3:

$$(x + 1) \log 3 = \log 80.5$$

Distribute log 3 and solve for x:

$$x \log 3 + \log 3 = \log 80.5$$
$$x \log 3 = \log 80.5 - \log 3$$
$$x = \frac{\log 80.5 - \log 3}{\log 3}$$

(Slide #14)

Exponential equations are used in finding compound interest. Here is the equation for this computation:

$$A = P[1 + (\tfrac{r}{n})]^{ny}$$

P is money invested,
r is interest rate per year,
n is the number of times the interest
is compounded per year,
y is the number of years of the investment,
and A is the total amount of money made.

(Slide #17)

There is one last bit of information about logarithms of various bases. A "change of base formula" can be used to change from any base to any other base.

$$\log_x y = \frac{\log_b y}{\log_b x}$$

(Slide #32)

Khan Academy helpful video: http://www.khanacademy.org/science/core-finance/interest-tutorial/compound-interest-tutorial/v/introduction-to-compound-interest

You should be able to solve the following problems prior to moving to the Practice Test:

Use logarithms to solve for x: $5^{x+1} = 24$

$x = \dfrac{\log 5}{\log 24 - \log 5}$

$x = \dfrac{\log 24 - \log 5}{\log 5}$

$x = \dfrac{\log 5}{\log 24}$

$x = \dfrac{\log 24}{\log 5}$

Use change-of-base formula to find three significant digits.

$\log_7 648$

You have $2500 to invest at 6% interest compounded quarterly. For how many years will the money need to be invested for that amount to triple?

$$A = P[1 + (\tfrac{r}{n})]^{ny}$$

Arithmetic Sequence (L 028)

The general form of an arithmetic sequence is

$a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots,$

where a_1 is the first term and d is the common difference.

An infinite arithmetic sequence has no last term. It just keeps going on forever. *(Slide #3)*

A finite arithmetic sequence has a last term: a_y

To get to this nth term, d is added to a_1 exactly n – 1 times.

Thus, $a_y = a_1 + (n - 1)d$. *(Slide #4)*

Khan Academy helpful video:
http://www.khanacademy.org/math/trigonometry/seq_induction/seq_and_series/v/arithmetic-sequences

You should be able to solve the following problems prior to moving to the Practice Test:

Find the three arithmetic means in this sequence.

12, ___, ___, ___, 40

Arithmetic Series (L 029)

Arithmetic Series

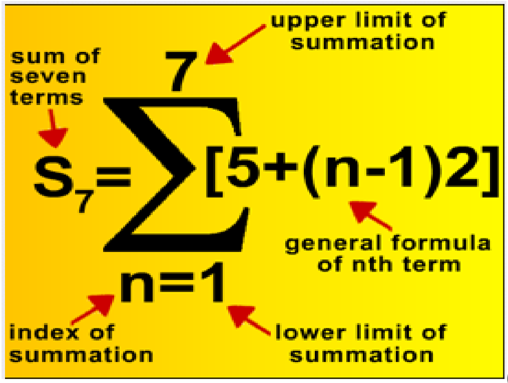
An arithmetic series is the indicated sum of the terms of an arithmetic sequence.

In other words, a sequence becomes a series when the terms are added rather than merely listed. *(Slide #1)*

There is a shorter way to write the sum using the Greek letter sigma.

$$\Sigma$$

Sigma is often called a summation sign. *(Slide #4)*



(Slide #7)

It seems, then, that we have a formula for finding the sum of an arithmetic series by using the first and last terms, $a_1 + a_y$, and the number of terms, n.

$$S_y = \frac{n}{2}(a_1 + a_y)$$

(Slide #20)

Khan Academy helpful video:

http://www.khanacademy.org/math/trigonometry/seq_induction/seq_and_series/v/sum-of-arithmetic-sequence--arithmetic-series

You should be able to solve the following problem prior to moving to the Practice Test:

Find a_n for the arithmetic series with $S_{16} = -288$, and $a_1 = -21$.

Geometric Sequence (L 030)

Geometric Sequences

A geometric sequence is one in which each term is found by multiplying the previous term by a constant number, r.

This constant is called the common ratio.

(Slide #1)

For the nth term, we have multiplied a_1 by r exactly n – 1 times. Thus,

$$a_y = a_1(r)^{x-1}.$$

(Slide #11)

COMPARE:

Geometric sequences have common ratios. One term is multiplied by this ratio to find the next term.

Arithmetic sequences have common differences. One term is added to this difference to find the next term.

(Slide #21)

Khan Academy helpful video:
http://www.khanacademy.org/math/trigonometry/seq_induction/seq_and_series/v/geometric-sequences--introduction

You should be able to solve the following problem prior to moving to the Practice Test:

For the geometric sequence with $a_1 = 7$ and $r = 2$, find a_5.

Geometric Series (L 031)

It can be shown that the sum of a finite geometric series is:

$$S_y = \frac{a_1(1 - r^x)}{1 - r}$$

where a_1 is the first term, r is the common ratio,
n is the number of terms, and S_y is the sum of the n terms.

(Slide #5)

This formula can be rewritten:

$$S_y = \frac{a_1 - a_y r}{1 - r}$$

(Slide #12)

The sum on the left is an arithmetic series, because each increasing n increases a_y by 2.
The sum on the right is a geometric series, because each increasing n multiplies a_y by 3.

$$\sum_{n=1}^{5} 2n-1 \qquad \sum_{n=1}^{5} 3^n$$

(Slide #20)

Khan Academy helpful video:
http://www.khanacademy.org/math/trigonometry/seq_induction/advanced-sequences-series/v/formula-for-finite-geometric-series

You should be able to solve the following problem prior to moving to the Practice Test:

Find the sum of the geometric series with $a_1 = 7$, $r = 2$, and $n = 14$.

Infinite Geometric Series (L 032)

Infinite Geometric Series

In the last lesson, we worked with finite geometric series. Now consider this series:

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} \dots$$

This is an infinite series because there is no last term. Can the sum of an infinite number of terms be found?

(Slide #1)

So, for $|r| < 1$, the sum of an infinite geometric series converges to $S = \frac{a_1}{(1 - r)}$.

When a series converges, it means there is a limit to the sum. It does not keep growing without bound.

(Slide #5)

What happens when $|r| \geq 1$? Consider this series:

$$S = 1 + 2 + 4 + 8 + 16 + \ldots .$$

These numbers are getting larger and larger, as will the sum. It will increase without bound, so there is no limit to the sum. A series such as this diverges.

(Slide #6)

$0.\overline{3}$.

This can be written as $S = 0.\overline{3} = 0.3 + 0.03 + 0.003 + \ldots$.

Here, $a_1 = 0.3$ and $r = 0.1$.

$$S = \frac{0.3}{(1 - 0.1)}$$

$$S = \frac{0.3}{0.9}$$

$$S = \frac{1}{3}$$

$\frac{1}{3}$ is the fractional form of $0.\overline{3}$.

(Slide #20)

You should be able to solve the following problems prior to moving to the Practice Test:

Choose the infinite geometric series of $.\overline{37}$.

- $.37 + .037 + .0037 + \ldots$
- $.37 + .007 + .0007 + \ldots$
- $.37 + .0037 + .000037 + \ldots$
- $.37 + .370 + .3700 + \ldots$

Find the value of r for an infinite geometric series with S = 96 and $a_1 = 12$.

$-\frac{7}{12}$	$-\frac{7}{8}$
$\frac{12}{8}$	$\frac{7}{8}$

Binomial Theorem (L 033)

n! Factorials n!

In order to use the binomial theorem, you must understand factorial notation. Factorials are a fast way to write repeated multiplication (and avoid writer's cramp).

If "n" is a positive integer, the expression n! (n factorial) is
$n! = n(n - 1)(n - 2) \ldots (2)(1)$
(By definition, 0! = 1)

(Slide #2)

Consider $\frac{12!}{6!}$ and its expanded form.

$$\frac{12!}{6!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

Because both numerator and denominator have 6!, the answer will be

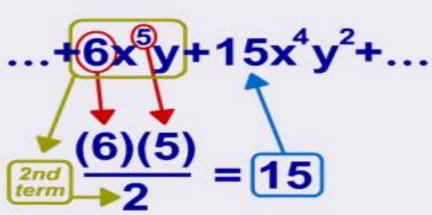
$$12 \times 11 \times 10 \times 9 \times 8 \times 7 = 665,280$$

(Slide #4)

It may help to think of the first term as containing y^0 and the last as having x^0. (Remember, anything to the power of zero will be one.)

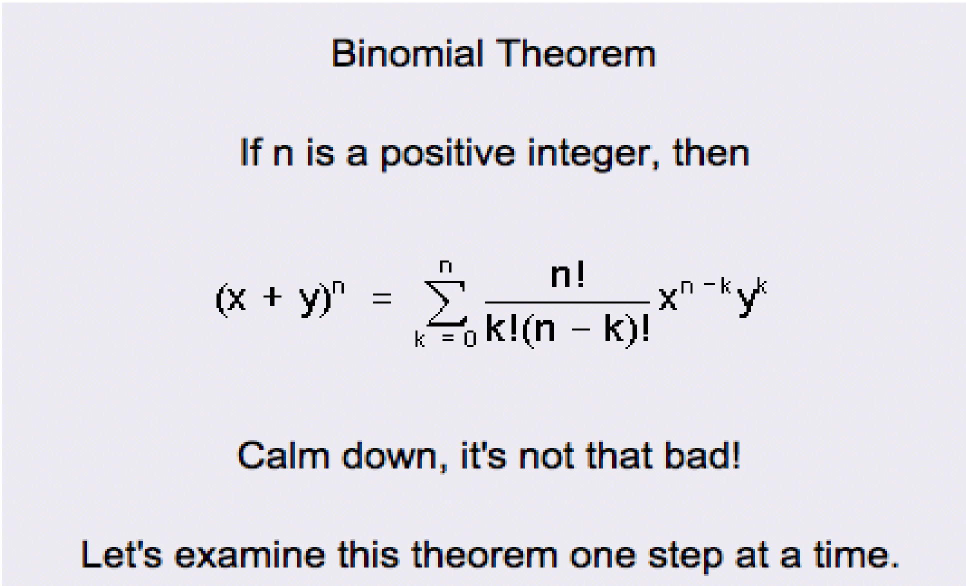
(Slide #12)

The coefficient of the third term is found by multiplying the second term's coefficient by the power of x in the second term.



Divide this product by the number of the term the factors came from (2). The result is the coefficient of the third term.

(Slide #16)



(Slide #31)

Khan Academy helpful videos:
http://www.khanacademy.org/math/trigonometry/polynomial_and_rational/binomial_theorem/v/binomial-theorem--part-1

AND

http://www.khanacademy.org/math/trigonometry/polynomial_and_rational/binomial_theorem/v/binomial-theorem--part-2

AND

http://www.khanacademy.org/math/trigonometry/polynomial_and_rational/binomial_theorem/v/binomial-theorem--part-3

You should be able to solve the following problem prior to moving to the Practice Test: